Supplementary Appendix to "Learning Your Comparative Advantages"

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1 Wage and Cutoffs Derivation

In this section we derive the solution to the value functions and the triggers.

For clarity, to distinguish between the case where the worker's outside option is his current occupation and the case where it is not (and consequently whether or not he is searching on the job), we introduce an additional subscript, k, to the three value functions and the wage function which denotes the worker's outside option. Formally we have

$$k = \arg\max_i \left[V_i(p) - U(p)\right].$$

J_{WB}, for example, denotes the asset value of a filled vacancy in occupation W with a worker whose outside option is occupation B, in other words when $p \in \left(\underline{p}, \widehat{p}\right)$. Similarly, $w_{BB}(p)$, denotes the wage of a worker who does not find it optimal to switch occupations by on the job search, thus $p \in [0, \widehat{p}]$, while U_W denotes the value of an unemployed worker searching for a job in occupation W, i.e. $p \in [\widehat{p}, 1]$.

The case where the worker's outside option is the occupation he is currently employed in, i.e. $i = k$, is analyzed in Appendix B to the paper.

For the case where the worker's occupation of choice if unemployed differs from his current one ($i \neq k$), equations (3), (B.1) and (B.2) in the paper now become

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$$
\begin{aligned}
rV_{ik}(p) &= w_{ik}(p) + \frac{1}{2}\zeta_i^2 p^2 (1-p)^2 V_{ik}''(p) \\
&\quad -\delta_i [V_{ik}(p) - U_k(p)] - \gamma V_{ik}(p) + \eta_i \lambda [V_{kk}(p) - V_{ik}(p)]
\end{aligned} \tag{3''}
$$

$$
\begin{aligned}
rJ_{ik}(p) &= \bar{a}_i(p) - w_{ik}(p) + \frac{1}{2}\zeta_i^2 p^2 (1-p)^2 J_{ik}''(p) \\
&\quad -\delta_i J_{ik}(p) - \gamma J_{ik}(p) - \eta_i \lambda J_{ik}(p)
\end{aligned} \tag{B.1''}
$$

$$
rU_k(p) = b_u + \lambda [V_{kk}(p) - U_k(p)] - \gamma U_k(p). \tag{B.2''}
$$

By subtracting (B.2") from (3") and multiplying through by $(1-q)$ and similarly multiplying (B.1") by q and then subtracting one from the other, we obtain the expression for the wage function

$$
\begin{aligned}
w_{ik}(p) &= q\bar{a}_i(p) + (1-q) b_u + q\lambda J_{kk}(p) \\
&\quad -\frac{1}{2}(1-q)\zeta_i^2 p^2 (1-p)^2 V_{ik}''(p) \\
&\quad +\frac{1}{2} q \zeta_i^2 p^2 (1-p)^2 J_{ik}''(p) \\
&\quad -q\eta_i \lambda J_{ik}(p) - (1-q)\eta_i \lambda (V_{kk}(p) - V_{ik}(p)).
\end{aligned} \tag{1A}
$$

Using the surplus sharing condition (eq. (2) in the paper), we have $V_{ik}''(p) = \frac{q}{1-q} J_{ik}''(p) + U_k''(p)$ and from the value of being unemployed we obtain $U_k''(p) = \frac{\lambda}{r+\gamma+\lambda} V_{kk}''(p)$. Therefore

$$
V_{ik}''(p) = \frac{q}{1-q} J_{ik}''(p) + \frac{\lambda}{r+\gamma+\lambda} V_{kk}''(p),
$$

which we use to substitute out for $V_{ik}''(p)$ in the wage equation (eq. (1A))

$$
\begin{aligned}
&w_{ik}(p) = q\bar{a}_i(p) + (1-q)\, b_u + q\lambda J_{kk}(p) \\
&-\frac{1}{2}(1-q)\,\zeta_i^2 p^2 (1-p)^2 \left(\frac{q}{1-q} J''_{ik}(p) + \frac{\lambda}{r+\gamma+\lambda} V''_{kk}(p)\right) \\
&+\frac{1}{2} q \zeta_i^2 p^2 (1-p)^2 J''_{ik}(p) \\
&-q\eta_i \lambda J_{ik}(p) - (1-q)\,\eta_i \lambda \left(V_{kk}(p) - V_{ik}(p)\right),
\end{aligned}
$$

which implies

$$
\begin{aligned}
w_{ik}(p) \;=\;& q\bar{a}_i(p) + (1-q)\, b_u + q\lambda J_{kk}(p) \\
&-\frac{\lambda(1-q)}{2(r+\gamma+\lambda)} \zeta_i^2 p^2 (1-p)^2 V''_{kk}(p) \\
&-q\eta_i \lambda J_{ik}(p) - (1-q)\,\eta_i \lambda \left(V_{kk}(p) - V_{ik}(p)\right).
\end{aligned}
\tag{2A}
$$

To obtain equation (B.6) in the paper, we once again make use of the equality $U''_k(p) = \frac{\lambda}{r+\gamma+\lambda} V''_{kk}(p)$. Moreover from the surplus sharing condition we have $q\left(J''_{ik}(p) + V''_{ik}(p)\right) - V''_{ik}(p) = -(1-q)\, U''_k(p)$. Using these two conditions, one can transform the wage equation above, to eq. (B.6) of the paper.

Once again, using the surplus sharing condition we have

$$
V_{kk}(p) = \frac{q}{1-q} J_{kk}(p) + U_k(p),
$$

and

$$
V_{ik}(p) = \frac{q}{1-q} J_{ik}(p) + U_k(p).
$$

Subtracting one from the other

$$
V_{kk}(p) - V_{ik}(p) = \frac{q}{1-q}\left(J_{kk}(p) - J_{ik}(p)\right).
$$

Using the above result as well as equation (B.5) of Appendix B of the main paper, allows us to write the wage equation (eq. (2A)) as function of J_{kk}

$$
w_{ik}(p) = q\bar{a}_i(p) + (1-q)\, b_u - \frac{q\lambda}{2(r+\gamma)} \zeta_i^2 p^2 (1-p)^2 J''_{kk}(p) + q\lambda(1-\eta_i)\, J_{kk}(p). \tag{3A}
$$

Substituting in for the wage (eq. (3A)) in the asset value of the firm condition gives us the following differential equation (eq. (B.8) in the paper)

$$\begin{aligned}
\left(r+\delta_i+\gamma+\eta_i\lambda\right) J_{ik}(p) &= \bar{a}_i(p) - q\bar{a}_i(p) - (1-q)\, b_u \\
&+\frac{q\lambda}{2(r+\gamma)}\zeta_i^2 p^2 (1-p)^2 J''_{kk}(p) - q\lambda\,(1-\eta_i)\, J_{kk}(p) \\
&+\frac{1}{2}\zeta_i^2 p^2 (1-p)^2 J''_{ik}(p)\,.
\end{aligned}$$

We can now use the expression we derived above and replace J_{kk} and J''_{kk} and obtain a differential equation in $J_{ik}(p)$. More specifically, for the case of $i = W$ and $k = B$, we have

$$\phi_{WB} p^2 (1-p)^2 J''_{WB}(p) = J_{WB}(p) + c_{WB} + \theta_{WB} p + \kappa_{WB} K_2^B p^{\xi_{WB}} (1-p)^{1-\xi_{WB}}\,,$$

where

$$\begin{aligned}
\phi_{WB} &= \frac{1}{2\left(r+\gamma+\delta_W+\eta_W\lambda\right)}\zeta_W^2 \\
c_{WB} &= \frac{1-q}{r+\gamma+\delta_W+\eta_W\lambda}\left(q\lambda\,(1-\eta_W)\frac{\left(a_B^b-b_u\right)}{r+\gamma+\delta_B+q\lambda} - \left(a_W^b-b_u\right)\right) \\
\theta_{WB} &= \frac{1-q}{r+\gamma+\delta_W+\eta_W\lambda}\left(q\lambda\,(1-\eta_W)\frac{a_B^w-a_B^b}{r+\gamma+\delta_B+q\lambda} - \left(a_W^w-a_W^b\right)\right) \\
\kappa_{WB} &= -\frac{q\lambda}{r+\gamma+\delta_W+\eta_W\lambda}\left(\frac{r+\gamma+\delta_B+q\lambda}{r+\gamma+q\lambda}\left(\frac{\zeta_W}{\zeta_B}\right)^2 - (1-\eta_W)\right) \\
\xi_{WB} &= \frac{1}{2}+\frac{1}{2}\sqrt{\frac{4+h_B}{h_B}} \\
h_B &= \frac{1}{2}\frac{r+\gamma+q\lambda}{\left(r+\gamma+\delta_B+q\lambda\right)(r+\gamma)}\zeta_B^2.
\end{aligned}$$

Similarly for the case of $i = B$ and $k = W$

$$\phi_{BW} p^2 (1-p)^2 J''_{BW}(p) = J_{BW}(p) + c_{BW} + \theta_{BW} p + \kappa_{BW} K_1^W p^{\xi_{BW}} (1-p)^{1-\xi_{BW}}\,,$$

where again

$$
\begin{aligned}
\phi_{BW} &= \frac{1}{2\left(r+\gamma+\delta_B+\eta_B\lambda\right)}\zeta_B^2 \\
c_{BW} &= \frac{1-q}{r+\gamma+\delta_B+\eta_B\lambda}\left(q\lambda\left(1-\eta_B\right)\frac{\left(a_W^b-b_u\right)}{r+\gamma+\delta_W+q\lambda}-\left(a_B^b-b_u\right)\right) \\
\theta_{BW} &= \frac{1-q}{r+\gamma+\delta_B+\eta_B\lambda}\left(q\lambda\left(1-\eta_B\right)\frac{a_W^w-a_W^b}{r+\gamma+\delta_W+q\lambda}-\left(a_B^w-a_B^b\right)\right) \\
\kappa_{BW} &= -\frac{q\lambda}{r+\gamma+\delta_B+\eta_B\lambda}\left(\frac{r+\gamma+\delta_W+q\lambda}{r+\gamma+q\lambda}\left(\frac{\zeta_B}{\zeta_W}\right)^2-\left(1-\eta_B\right)\right) \\
\xi_{BW} &= \frac{1}{2}-\frac{1}{2}\sqrt{\frac{4+h_W}{h_W}} \\
h_W &= \frac{1}{2}\frac{r+\gamma+q\lambda}{\left(r+\gamma+\delta_W+q\lambda\right)\left(r+\gamma\right)}\zeta_W^2.
\end{aligned}
$$

Using the value matching and smooth pasting conditions ($J_{WB}\left(\underline{p}\right)=0$ and $J'_{WB}\left(\underline{p}\right)=0$ for the case of $i=W$ and $J_{BW}\left(\overline{p}\right)=0$ and $J'_{BW}\left(\overline{p}\right)=0$ for the case of $i=B$) we solve the above differential equations to obtain equations (B.9) and (B.10) of the paper.

Finally, to pin down the values of $\underline{p}$, $\overline{p}$, $\widehat{p}$, K_1^W and K_2^B we use the 5 conditions mentioned in Appendix B to the main paper. In particular optimality of search behavior implies that $V_W\left(\widehat{p}\right)=V_B\left(\widehat{p}\right)$. The remaining 4 conditions are given by the continuity of the total value of the match, V_i+J_i at $\widehat{p}$, as well as continuity of $V_i'+J_i'$ at $\widehat{p}$.[1] We prove the validity of these 4 conditions next.

Denote the total value of the match by $H_i\left(p\right)\equiv V_i\left(p\right)+J_i\left(p\right)$.

From our setup the total value of the match is given by the following Hamilton-Jacobi-Bellman equation

$$
H_i\left(p\right)=\overline{a}_i\left(p\right)-\delta_i\left(H_i\left(p\right)-U\left(p\right)\right)-\gamma H_i\left(p\right)+\frac{1}{2}\zeta_i^2p^2\left(1-p\right)^2H_i''\left(p\right).
$$

The expected flow output of the match is given by $\overline{a}_i\left(p\right)$, the match is dissolved exogenously at rate δ_i, in which case the worker becomes unemployed and obtains value $U\left(p\right)$, while the firm becomes vacant . If the worker retires exogenously, both the worker's and the firm's value equals zero. The last term on the right hand side captures the match's option value of learning.

We know that optimal match dissolution implies a value matching and a smooth pasting condition at $\underline{p}$ when $i=W$ (and at $\overline{p}$ when $i=B$). In other words, $H_W\left(\underline{p}\right)=U\left(\underline{p}\right)$ and

[1] I am grateful to Björn Brügemann for his input in this part.

$H'_W\left(\underline{p}\right)=U'\left(\underline{p}\right)$. Furthermore we know that $U\left(p\right)$ is continuous (follows from eq. (B.2) in the paper and $V_W\left(\widehat{p}\right)=V_B\left(\widehat{p}\right)$), but that it may have kinks. We show that H_i is nevertheless continuously differentiable at $\widehat{p}$ despite the presence of a possible kink at $U\left(\widehat{p}\right)$.

The solution to the above differential equation when $i=W$ is given by

$$\begin{aligned}
H_W\left(p\right) &= \frac{1}{1-2\theta_W}[p^{\theta_W}\left(1-p\right)^{1-\theta_W} \\
&\quad (\underline{p}^{-\theta_W}\left(1-\underline{p}\right)^{\theta_W-1}\left(1-\theta_W-\underline{p}\right)U\left(\underline{p}\right)-\underline{p}^{1-\theta_W}\left(1-\underline{p}\right)^{\theta_W}U'\left(\underline{p}\right) \\
&\quad -\frac{2}{\zeta_i^2}\int_{\underline{p}}^{p}\left(\overline{a}_W\left(\tau\right)+\delta U\left(\tau\right)\right)\left(-\tau^{-1-\theta_W}\left(1-\tau\right)^{\theta_W-2}\right)d\tau) \\
&\quad +p^{1-\theta_W}\left(1-p\right)^{\theta_W} \\
&\quad (-\underline{p}^{\theta_W-1}\left(1-\underline{p}\right)^{-\theta_W}\left(p-\underline{p}\right)U\left(\underline{p}\right)+\underline{p}^{\theta_W}\left(1-\underline{p}\right)^{1-\theta_W}U'\left(\underline{p}\right) \\
&\quad -\frac{2}{\zeta_i^2}\int_{\underline{p}}^{p}\left(\overline{a}_W\left(\tau\right)+\delta U\left(\tau\right)\right)\left(\tau^{\theta_W-2}\left(1-\tau\right)^{-1-\theta_W}\right)d\tau)],
\end{aligned}$$

where $\theta_i\equiv\frac{\zeta_i-\sqrt{8(r+\delta+\gamma)+\zeta_i^2}}{2\zeta_i}$ and it is clearly continuous. The solution to H_B is identical.

Differentiating the value of the match above yields

$$\begin{aligned}
H'_W\left(p\right) &= \frac{1}{1-2\theta_W}[\left(p^{\theta_W-1}\left(1-p\right)^{-\theta_W}\left(\theta_W-p\right)\right) \\
&\quad (\underline{p}^{-\theta_W}\left(1-\underline{p}\right)^{\theta_W-1}\left(1-\theta_W-\underline{p}\right)U\left(\underline{p}\right)-\underline{p}^{1-\theta_W}\left(1-\underline{p}\right)^{\theta_W}U'\left(\underline{p}\right) \\
&\quad -\frac{2}{\zeta_i^2}\int_{\underline{p}}^{p}\left(\overline{a}_W\left(\tau\right)+\delta U\left(\tau\right)\right)\left(-\tau^{-1-\theta_W}\left(1-\tau\right)^{\theta_W-2}\right)d\tau) \\
&\quad +\left(p^{-\theta_W}\left(1-p\right)^{\theta_W-1}\left(1-p-\theta_W\right)\right) \\
&\quad (-\underline{p}^{\theta_W-1}\left(1-\underline{p}\right)^{-\theta_W}\left(\theta_W-\underline{p}\right)U\left(\underline{p}\right)+\underline{p}^{\theta_W}\left(1-\underline{p}\right)^{1-\theta_W}U'\left(\underline{p}\right) \\
&\quad -\frac{2}{\zeta_i^2}\int_{\underline{p}}^{p}\left(\overline{a}_W\left(\tau\right)+\delta U\left(\tau\right)\right)\left(\tau^{\theta_W-2}\left(1-\tau\right)^{-1-\theta_W}\right)d\tau)],
\end{aligned}$$

which is also continuous.

Moreover, using equations (B.11) and (B.13) of the main paper we can solve for K_1^W and K_2^B as function of $\underline{p}$, $\overline{p}$ and $\widehat{p}$. In particular we obtain

$$K_1^W=\frac{L}{M}+\frac{Q}{M-GQ}\left(F+\frac{GL}{M}\right)$$

$$K_2^B=\frac{GL+MF}{M-GQ},$$

where

$$M=\widehat{p}^{\xi_{BW}}\left(1-\widehat{p}\right)^{1-\xi_{BW}}$$

$$\begin{aligned}
L &= \frac{1}{\sqrt{\phi_{WB}\left(4+\phi_{WB}\right)}}\widehat{p}^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\widehat{p}\right)^{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}} \\
&\int_{\underline{p}}^{\widehat{p}}\left[\theta_{WB}\tau+c_{WB}\right]\left(-\tau^{-\frac{3}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\tau\right)^{-\frac{3}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\right)d\tau \\
&+\frac{1}{\sqrt{\phi_{WB}\left(4+\phi_{WB}\right)}}\widehat{p}^{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\widehat{p}\right)^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}} \\
&\int_{\underline{p}}^{\widehat{p}}\left[\theta_{WB}\tau+c_{WB}\right]\left(\tau^{-\frac{3}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\tau\right)^{-\frac{3}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\right)d\tau \\
&-\frac{\left(1-q\right)\left(a_W^w-a_W^b\right)}{r+\gamma+\delta_W+q\lambda}\widehat{p}-\frac{\left(1-q\right)\left(a_W^b-b\right)}{r+\gamma+\delta_W+q\lambda}
\end{aligned}$$

$$\begin{aligned}
Q &= \frac{1}{\sqrt{\phi_{WB}\left(4+\phi_{WB}\right)}}\widehat{p}^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\widehat{p}\right)^{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}} \\
&\int_{\underline{p}}^{\widehat{p}}\left[\kappa_{WB}\tau^{\xi_{WB}}\left(1-\tau\right)^{1-\xi_{WB}}\right]\left(-\tau^{-\frac{3}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\tau\right)^{-\frac{3}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\right)d\tau \\
&+\frac{1}{\sqrt{\phi_{WB}\left(4+\phi_{WB}\right)}}\widehat{p}^{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\widehat{p}\right)^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}} \\
&\int_{\underline{p}}^{\widehat{p}}\left[\kappa_{WB}\tau^{\xi_{WB}}\left(1-\tau\right)^{1-\xi_{WB}}\right]\left(\tau^{-\frac{3}{2}-\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\left(1-\tau\right)^{-\frac{3}{2}+\frac{1}{2}\sqrt{\frac{4+\phi_{WB}}{\phi_{WB}}}}\right)d\tau
\end{aligned}$$

$$\begin{aligned}
F &= \frac{1}{\widehat{p}^{\xi_{WB}}\left(1-\widehat{p}\right)^{1-\xi_{WB}}}\frac{\left(1-q\right)\left(a_W^w-a_W^b\right)}{r+\gamma+\delta_W+q\lambda}\widehat{p} \\
&+\frac{1}{\widehat{p}^{\xi_{WB}}\left(1-\widehat{p}\right)^{1-\xi_{WB}}}\frac{\left(1-q\right)\left(a_W^b-b\right)}{r+\gamma+\delta_W+q\lambda} \\
&-\frac{1}{\widehat{p}^{\xi_{WB}}\left(1-\widehat{p}\right)^{1-\xi_{WB}}}\frac{\left(1-q\right)\left(a_B^w-a_B^b\right)}{r+\gamma+\delta_B+q\lambda}\widehat{p} \\
&-\frac{1}{\widehat{p}^{\xi_{WB}}\left(1-\widehat{p}\right)^{1-\xi_{WB}}}\frac{\left(1-q\right)\left(a_B^b-b\right)}{r+\gamma+\delta_B+q\lambda}
\end{aligned}$$

$$G = \frac{\widehat{p}^{\xi_{BW}} \left(1-\widehat{p}\right)^{1-\xi_{BW}}}{\widehat{p}^{\xi_{WB}} \left(1-\widehat{p}\right)^{1-\xi_{WB}}}.$$

Thus K_1^W and K_2^B are uniquely pinned down, given the values of $\underline{p}$, $\overline{p}$, $\widehat{p}$. However, as we show in Appendix B of the paper $\underline{p}$, $\overline{p}$, $\widehat{p}$ all have unique solutions, so K_1^W and K_2^B also have unique solutions.

2 Endogenizing the Job-Finding Rate

In order to close the model we need to endogenize the worker's job-finding rate λ_i. We assume there is a large mass of ex ante homogeneous firms ensuring free entry in each occupation. In each occupation i, matches are randomly formed according to an increasing, concave and homogenous of degree one matching function $m(p_i, v_i)$, where p_i is the effective mass of workers (employed or unemployed) petitioning for a job in occupation i and v_i is the mass of occupation i vacancies. Unemployed workers are matched with firms at rate $\lambda_i = \frac{m(p_i,v_i)}{p_i}$. Each firm can post a vacancy at flow cost cv_i, which earns no revenue while empty.

Since employed workers contact firms at a different rate than unemployed workers, the effective mass of workers searching for a job in each occupation is equal to

$$\begin{aligned} p_W &= \int_{\widehat{p}}^{1} z_W(p)\,dp + \eta_B \int_{\widehat{p}}^{\overline{p}} f_B(p)\,dp \\ p_B &= \int_{0}^{\widehat{p}} z_B(p)\,dp + \eta_W \int_{\underline{p}}^{\widehat{p}} f_W(p)\,dp. \end{aligned}$$

A firm in occupation W contacts a worker at rate $\frac{m(p_W,v_W)}{v_W}$, so the value of an unfilled vacancy in occupation W, P_W equals

$$P_W = -cv_W + \frac{m(p_W, v_W)}{v_W} \int_{\widehat{p}}^{1} J_W(p)\, z_W(p)\,dp + \frac{m(p_W, v_W)}{v_W} \eta_B \int_{\widehat{p}}^{\overline{p}} J_W(p)\, f_B(p)\,dp.$$

Therefore the asset value of a vacancy is equal to its flow cost plus the expected value of filled posting, when it contacts a worker. Note that because on the job search has a different effectiveness, the effective mass of potential workers is η_i times the mass of the employed searchers.

Similarly for occupation B

$$P_B = -cv_B + \frac{m(p_B, v_B)}{v_B} \int_{0}^{\widehat{p}} J_B(p)\, z_B(p)\,dp + \frac{m(p_B, v_B)}{v_B} \eta_W \int_{\underline{p}}^{\widehat{p}} J_B(p)\, f_W(p)\,dp.$$

Free firm entry and exit ensures that $P_i = 0$ and therefore

$$cv_W = \frac{m\left(p_W, v_W\right)}{v_W}\int_{\widehat{p}}^{1} J_W\left(p\right) z_W\left(p\right) dp + \frac{m\left(p_W, v_W\right)}{v_W}\eta_B \int_{\widehat{p}}^{\overline{p}} J_W\left(p\right) f_B\left(p\right) dp,$$

and

$$cv_B = \frac{m\left(p_B, v_B\right)}{v_B}\int_{0}^{\widehat{p}} J_B\left(p\right) z_B\left(p\right) dp + \frac{m\left(p_B, v_B\right)}{v_B}\eta_W \int_{\underline{p}}^{\widehat{p}} J_B\left(p\right) f_W\left(p\right) dp.$$

In the derivations of the preceding sections, we have assumed that $\lambda_W = \lambda_B = \lambda$ to simplify the analysis. Given the estimated value of λ, the above two equations uniquely pin down the implied vacancy costs cv_i.

3 Aggregation

3.1 Steady-State Distribution

As mentioned in the paper, the "birth" distribution, $g\left(p\right)$ follows a $beta(\psi_1, \psi_2)$, so that $g\left(p\right) = \frac{1}{B(\psi_1,\psi_2)} p^{\psi_1 - 1}\left(1-p\right)^{\psi_2 - 1}$ where $B\left(\psi_1, \psi_2\right) = \int_0^1 x^{\psi_1-1}\left(1-x\right)^{\psi_2-1} dx$ with $\psi_1, \psi_2 > 0$ denotes the beta function[2]. The mean value of the distribution also pins down the true percentage of white type workers in the economy.

In Appendix C of the paper we derive the following system of differential with respect to the population density of employed workers in each occupation

$$\frac{d^2}{dp^2}\left[\frac{1}{2}\zeta_W^2 p^2\left(1-p\right)^2 f_W\left(p\right)\right] - \gamma\frac{\delta_W + \lambda + \gamma}{\lambda + \gamma} f_W\left(p\right) + \frac{\lambda\delta_B}{\lambda+\gamma} f_B\left(p\right) + \frac{\gamma\lambda}{\lambda+\gamma} g\left(p\right) - \eta_W \lambda f_W\left(p\right) I\left\{p < \widehat{p}\right\} + \eta_B \lambda f_B\left(p\right) I\left\{\widehat{p} \leq p \leq \overline{p}\right\} = 0 \tag{4A}$$

$$\frac{d^2}{dp^2}\left[\frac{1}{2}\zeta_B^2 p^2\left(1-p\right)^2 f_B\left(p\right)\right] - \gamma\frac{\delta_B + \lambda + \gamma}{\lambda + \gamma} f_B\left(p\right) + \frac{\lambda\delta_W}{\lambda+\gamma} f_W\left(p\right) + \frac{\gamma\lambda}{\lambda+\gamma} g\left(p\right) - \eta_B \lambda f_B\left(p\right) I\left\{\widehat{p} \leq p\right\} + \eta_W \lambda f_W\left(p\right) I\left\{\underline{p} \leq p < \widehat{p}\right\} = 0. \tag{5A}$$

Now, in order to solve the above system of differential equations, (4A) and (5A), we have to take cases.

[2]The population density from which the posterior is drawn is equal to $g\left(p\right) M$, where M is the total mass of workers, here normalized to 1.

For example, for $p \in [0, \underline{p})$, we know that $f_W(p) = 0$ (workers in occupation W quit once they hit $\underline{p}$), so equation (5A) reduces to

$$\frac{d^2}{dp^2}\left[\frac{1}{2}\zeta_B^2 p^2 (1-p)^2 f_B(p)\right] - \gamma\frac{\delta_B+\lambda+\gamma}{\lambda+\gamma} f_B(p) + \frac{\lambda\gamma}{\lambda+\gamma} g(p) = 0.$$

The solution to the above differential equation is given by

$$\begin{aligned} f_B(p) &= C_1^B p^{q_B-2}(1-p)^{-1-q_B} + C_2^B p^{-1-q_B}(1-p)^{q_B-2} \\ &\quad - \frac{d_B}{\sqrt{c_B(4+c_B)}} p^{q_B-2}(1-p)^{-1-q_B}\int_0^p \tau^{\psi_1-q_B}(1-\tau)^{q_B+\psi_2-1}\,d\tau \\ &\quad + \frac{d_B}{\sqrt{c_B(4+c_B)}} p^{-1-q_B}(1-p)^{q_B-2}\int_0^p \tau^{q_B+\psi_1-1}(1-\tau)^{\psi_2-q_B}\,d\tau, \end{aligned}$$

where $c_B = \frac{\zeta_B^2(\lambda+\gamma)}{2\gamma(\delta_B+\lambda+\gamma)}$, $q_B = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{4+c_B}{c_B}}$, $d_B = -\frac{\lambda}{(\delta_B+\lambda+\gamma)B(\psi_1,\psi_2)}$ and C_1^B and C_2^B are undetermined coefficients. However since $\int_x^p p^{q_B-2}dp = \frac{1}{q_B-1}p^{q_B-1}|_x^p = +\infty$ as $x \to 0$, because $q_B < 0$, it must be the case that $C_1^B = 0$ because the mass of workers at any interval is bounded from above by one. For the same reason we require that $q_B + \psi_1 > 0$.

For $p \in (\underline{p}, \widehat{p})$, for the case of occupation W, since $z_W(p) = 0$ at that interval, equation (C.16) of the paper becomes

$$\frac{d^2}{dp^2}\left[\frac{1}{2}\zeta_W^2 p^2 (1-p)^2 f_W(p)\right] - (\delta_W+\gamma) f_W(p) - \eta_W \lambda f_W(p) = 0.$$

The solution to the above differential equation is given by

$$f_W(p) = C_5^W p^{-\frac{3}{2}-\kappa_B}(1-p)^{-\frac{3}{2}+\kappa_B} + C_6^W p^{-\frac{3}{2}+\kappa_B}(1-p)^{-\frac{3}{2}-\kappa_B},$$

where $\kappa_B = \sqrt{\frac{1}{4} + \frac{2(\delta_W+\gamma+\eta_W\lambda)}{\zeta_W^2}}$.

Using the boundary condition, $f_W(\underline{p}) = 0$, we are able to substitute out for C_5^W.

We can now substitute out for $f_W(p)$ in equation (5A) and obtain

$$\begin{aligned}
& \frac{d^2}{dp^2}\left[\frac{1}{2}\zeta_B^2 p^2 (1-p)^2 f_B(p)\right] - \gamma\frac{\delta_B+\lambda+\gamma}{\lambda+\gamma} f_B(p) \\
& -C_6^W\left(\frac{\lambda\delta_W}{\lambda+\gamma}+\eta_W\lambda\right)\left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B} p^{-\frac{3}{2}-\kappa_B}(1-p)^{-\frac{3}{2}+\kappa_B} \\
& +C_6^W\left(\frac{\lambda\delta_W}{\lambda+\gamma}+\eta_W\lambda\right) p^{-\frac{3}{2}+\kappa_B}(1-p)^{-\frac{3}{2}-\kappa_B} \\
& +\frac{\lambda\gamma}{\lambda+\gamma}\frac{1}{B(\psi_1,\psi_2)} p^{\psi_1-1}(1-p)^{\psi_2-1} \\
= \; & 0.
\end{aligned}$$

The solution to the above differential equation gives the expression for $f_B(p)$ for the $(\underline{p}, \widehat{p})$ interval.

The derivation of $f_W(p)$ and $f_B(p)$ for the $[\widehat{p}, \overline{p})$ interval is symmetric and otherwise identical to the one for the $(\underline{p}, \widehat{p})$ interval: using the fact that $z_B(p) = 0$, equation (C.17) of the paper simplifies considerably and after solving the resulting differential equation and using the condition that $f_B(\overline{p}) = 0$, one obtains an expression for $f_B(p)$ with only one undetermined coefficient. Substituting into equation (4A) and solving, results in the corresponding expression for $f_W(p)$.

Finally, the case where $p \in (\overline{p}, 1]$ is similarly symmetric to the case where $p \in [0, \underline{p})$. Now we know that $f_B(p) = 0$ and therefore equation (4A) simplifies substantially. Solving the resulting differential equation and using the condition that the mass of employed workers in occupation W is bounded from above by one, obtains the expression for $f_W(p)$ in that interval with one undetermined coefficient (under the restriction that $q_W + \psi_2 > 0$).

Given the above, the resulting steady-state distribution of workers employed in occupation W is given by:

For $p \in (\underline{p}, \widehat{p}]$

$$f_W(p) = C_6^W (p(1-p))^{-\frac{3}{2}}\left(\left(\frac{p}{1-p}\right)^{\kappa_B} - \left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B}\left(\frac{1-p}{p}\right)^{\kappa_B}\right), \tag{6A}$$

for $p \in (\widehat{p}, \overline{p})$

$$
\begin{aligned}
f_W(p) &= C_3^W p^{q_W-2}(1-p)^{-1-q_W} + C_4^W p^{-1-q_W}(1-p)^{q_W-2} \qquad (7A) \\
&- \frac{1}{\sqrt{c_W(4+c_W)}} p^{q_W-2}(1-p)^{-1-q_W} [C_5^B m_W \int_{\widehat{p}}^{p} \tau^{-\frac{1}{2}+\kappa_W-q_W}(1-\tau)^{-\frac{3}{2}-\kappa_W+q_W} d\tau \\
&+ C_5^B n_W \int_{\widehat{p}}^{p} \tau^{-\frac{1}{2}-\kappa_W-q_W}(1-\tau)^{-\frac{3}{2}+\kappa_W+q_W} d\tau + d_W \int_{\widehat{p}}^{p} \tau^{\psi_1-q_W}(1-\tau)^{\psi_2+q_W-1} d\tau] \\
&+ \frac{1}{\sqrt{c_W(4+c_W)}} p^{-1-q_W}(1-p)^{q_W-2} [C_5^B m_W \int_{\widehat{p}}^{p} \tau^{-\frac{3}{2}+\kappa_W+q_W}(1-\tau)^{-\frac{1}{2}-\kappa_W-q_W} d\tau \\
&+ C_5^B n_W \int_{\widehat{p}}^{p} \tau^{-\frac{3}{2}-\kappa_W+q_W}(1-\tau)^{-\frac{1}{2}+\kappa_W-q_W} d\tau + d_W \int_{\widehat{p}}^{p} \tau^{\psi_1+q_W-1}(1-\tau)^{\psi_2-q_W} d\tau],
\end{aligned}
$$

and for $p \in [\overline{p}, 1]$

$$
\begin{aligned}
f_W(p) &= C_1^W p^{q_W-2}(1-p)^{-1-q_W} \qquad (8A) \\
&+ \frac{d_W}{\sqrt{c_W(4+c_W)}} p^{q_W-2}(1-p)^{-1-q_W} \int_p^1 \tau^{\psi_1-q_W}(1-\tau)^{q_W+\psi_2-1} d\tau \\
&- \frac{d_W}{\sqrt{c_W(4+c_W)}} p^{-1-q_W}(1-p)^{q_W-2} \int_p^1 \tau^{q_W+\psi_1-1}(1-\tau)^{\psi_2-q_W} d\tau,
\end{aligned}
$$

and similarly for occupation B, we obtain expressions (C.20)-(C.22) in the paper, where C_2^B, C_3^B, C_4^B, C_5^B, C_1^W, C_3^W, C_4^W and C_6^W are undetermined coefficients and

$$
\begin{aligned}
d_W &= -\frac{\lambda}{(\delta_W+\lambda+\gamma) B(\psi_1,\psi_2)} \\
d_B &= -\frac{\lambda}{(\delta_B+\lambda+\gamma) B(\psi_1,\psi_2)}
\end{aligned}
$$

$$
\begin{aligned}
n_W &= -\lambda \frac{\delta_B+\eta_B\lambda+\eta_B\gamma}{\gamma(\delta_W+\lambda+\gamma)} \\
n_B &= -\lambda \frac{\delta_W+\eta_W\lambda+\eta_W\gamma}{\gamma(\delta_B+\lambda+\gamma)}
\end{aligned}
$$

$$
\begin{aligned}
c_W &= \frac{\zeta_W^2(\lambda+\gamma)}{2\gamma(\delta_W+\lambda+\gamma)} \\
c_B &= \frac{\zeta_B^2(\lambda+\gamma)}{2\gamma(\delta_B+\lambda+\gamma)}
\end{aligned}
$$

$$\kappa_W = \sqrt{\frac{1}{4} + \frac{2\left(\delta_B + \gamma + \eta_B \lambda\right)}{\zeta_B^2}}$$
$$\kappa_B = \sqrt{\frac{1}{4} + \frac{2\left(\delta_W + \gamma + \eta_W \lambda\right)}{\zeta_W^2}}$$

$$m_W = -n_W \left(\frac{1-\overline{p}}{\overline{p}}\right)^{2\kappa_W}$$
$$m_B = -n_B \left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B}$$

$$q_W = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{4+c_W}{c_W}}$$
$$q_B = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{4+c_B}{c_B}}.$$

For the 8 undetermined coefficients we make use of the following 8 conditions.

Total flows in and out of employment in occupation W, must equal in the steady-state and therefore

$$\begin{aligned} & \lambda \int_{\widehat{p}}^{\overline{p}} z_W(p)\, dp + \lambda \int_{\overline{p}}^{1} z_W(p)\, dp + \lambda z_W(\overline{p}) + \eta_B \lambda \int_{\widehat{p}}^{\overline{p}} f_B(p)\, dp \\ = \; & \delta_W \int_{\underline{p}}^{1} f_W(p)\, dp + \gamma \int_{\underline{p}}^{1} f_W(p)\, dp + \frac{1}{2}\zeta_W^2 \underline{p}^2 \left(1-\underline{p}\right)^2 f_W'\left(\underline{p}+\right) + \eta_W \lambda \int_{\underline{p}}^{\widehat{p}} f_W(p)\, dp. \end{aligned} \tag{9A}$$

The first three terms on the left hand side, capture the inflow of workers from unemployment into employment, at rate λ. At $p = \overline{p}$, the endogenous exit of workers from occupation B, generates an atom of unemployed workers at that point. The fourth term captures the inflow of workers directly from occupation B. The first two terms on the right hand side denote the exit of workers due to endogenous match destruction and retirement respectively. The third term captures the endogenous exit of workers at $p = \underline{p}$, while the last term accounts for employed workers who find a job in occupation B.

The inflow of unemployed workers at $p = \overline{p}$ into employment in occupation W, $\lambda z_W(\overline{p})$, creates a kink rather than a discontinuity in the distribution of of employed workers: although there is an increased inflow of workers at $p = \overline{p}$, compared to the every other point of the distribution, as soon as new workers find a job and produce output, they start updating their beliefs, moving to points to the right and left of $\overline{p}$ and therefore preserving continuity

$$\lim_{\varepsilon \longrightarrow 0} f_W\left(\overline{p}+\varepsilon\right) = f_W\left(\overline{p}\right). \tag{10A}$$

Finally there is no reason for there to be a discontinuity in the level or the first derivative of f_W at $\widehat{p}$, so[3]

$$\lim_{\varepsilon \longrightarrow 0} f_W\left(\widehat{p}+\varepsilon\right) = f_W\left(\widehat{p}\right) \tag{11A}$$

$$\lim_{\varepsilon \longrightarrow 0} f'_W\left(\widehat{p}+\varepsilon\right) = f'_W\left(\widehat{p}\right). \tag{12A}$$

The above 4 conditions (equations (9A) through(12A)), along with the corresponding 4 conditions for occupation B, constitute a linear system of 8 equations and 8 unknowns that complete the solution to the steady distribution.

Therefore the remaining 8 undetermined coefficients are given by the solution to the linear system, which can be written in matrix form as follows

$$AC = B,$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 & a_{18} \\ 0 & 0 & 0 & a_{24} & a_{25} & a_{26} & a_{27} & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{46} & a_{47} & a_{48} \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} & a_{57} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & 0 & 0 & a_{68} \\ 0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} & a_{78} \\ 0 & a_{82} & a_{83} & a_{84} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C' = \begin{bmatrix} C_2^B & C_3^B & C_4^B & C_5^B & C_1^W & C_3^W & C_4^W & C_6^W \end{bmatrix}$$

$$B' = \begin{bmatrix} b_1 & b_2 & 0 & 0 & b_5 & b_6 & 0 & 0 \end{bmatrix},$$

and

[3] Unemployed workers no longer search in occupation W when the posterior crosses $\widehat{p}$ from above, but this creates a discontinuity in the second derivative of f_W rather than in the first, as is the case at $\overline{p}$.

$$
\begin{aligned}
a_{11} &= \underline{p}^{-1-q_B}\left(1-\underline{p}\right)^{q_B-2} \\
a_{12} &= -\underline{p}^{q_B-2}\left(1-\underline{p}\right)^{-1-q_B} \\
a_{13} &= -a_{11} \\
a_{18} &= \frac{1}{\sqrt{c_B\left(4+c_B\right)}}[\underline{p}^{q_B-2}\left(1-\underline{p}\right)^{-1-q_B}(m_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{1}{2}-\kappa_B-q_B}\left(1-\tau\right)^{-\frac{3}{2}+\kappa_B+q_B}d\tau \\
&\quad +n_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{1}{2}+\kappa_B-q_B}\left(1-\tau\right)^{-\frac{3}{2}-\kappa_B+q_B}d\tau) \\
&\quad -\underline{p}^{-1-q_B}\left(1-\underline{p}\right)^{q_B-2}(m_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{3}{2}-\kappa_B+q_B}\left(1-\tau\right)^{-\frac{1}{2}+\kappa_B-q_B}d\tau \\
&\quad +n_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{3}{2}+\kappa_B+q_B}\left(1-\tau\right)^{-\frac{1}{2}-\kappa_B-q_B}d\tau)]
\end{aligned}
$$

$$
\begin{aligned}
a_{24} &= \frac{1}{\sqrt{c_W\left(4+c_W\right)}}[\overline{p}^{q_W-2}\left(1-\overline{p}\right)^{-1-q_W}(m_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{1}{2}+\kappa_W-q_W}\left(1-\tau\right)^{-\frac{3}{2}-\kappa_W+q_W}d\tau \\
&\quad +n_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{1}{2}-\kappa_W-q_W}\left(1-\tau\right)^{-\frac{3}{2}+\kappa_W+q_W}d\tau) \\
&\quad -\overline{p}^{-1-q_W}\left(1-\overline{p}\right)^{q_W-2}(m_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{3}{2}+\kappa_W+q_W}\left(1-\tau\right)^{-\frac{1}{2}-\kappa_W-q_W}d\tau \\
&\quad +n_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{3}{2}-\kappa_W+q_W}\left(1-\tau\right)^{-\frac{1}{2}+\kappa_W-q_W}d\tau)] \\
a_{25} &= \overline{p}^{q_W-2}\left(1-\overline{p}\right)^{-1-q_W} \\
a_{26} &= -a_{25} \\
a_{27} &= -\overline{p}^{-1-q_W}\left(1-\overline{p}\right)^{q_W-2}
\end{aligned}
$$

$$
\begin{aligned}
a_{32} &= \widehat{p}^{q_B-2}\left(1-\widehat{p}\right)^{-1-q_B} \\
a_{33} &= \widehat{p}^{-1-q_B}\left(1-\widehat{p}\right)^{q_B-2} \\
a_{34} &= \left(\frac{1-\overline{p}}{\overline{p}}\right)^{2\kappa_W}\widehat{p}^{-\frac{3}{2}+\kappa_W}\left(1-\widehat{p}\right)^{-\frac{3}{2}-\kappa_W}-\widehat{p}^{-\frac{3}{2}-\kappa_W}\left(1-\widehat{p}\right)^{-\frac{3}{2}+\kappa_W}
\end{aligned}
$$

$$
\begin{aligned}
a_{46} &= \widehat{p}^{q_W-2}\left(1-\widehat{p}\right)^{-1-q_W} \\
a_{47} &= \widehat{p}^{-1-q_W}\left(1-\widehat{p}\right)^{q_W-2} \\
a_{48} &= \left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B}\widehat{p}^{-\frac{3}{2}-\kappa_B}\left(1-\widehat{p}\right)^{-\frac{3}{2}+\kappa_B}-\widehat{p}^{-\frac{3}{2}+\kappa_B}\left(1-\widehat{p}\right)^{-\frac{3}{2}-\kappa_B}
\end{aligned}
$$

$$
\begin{aligned}
a_{54} &= \frac{1}{2}\zeta_W^2\frac{1}{\sqrt{c_W\left(4+c_W\right)}}\cdot \\
&\quad ((-1-q_W+3\overline{p})\,\overline{p}^{-q_W}\left(1-\overline{p}\right)^{q_W-1} \\
&\quad \cdot[m_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{3}{2}+\kappa_W+q_W}\left(1-\tau\right)^{-\frac{1}{2}-\kappa_W-q_W}d\tau+n_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{3}{2}-\kappa_W+q_W}\left(1-\tau\right)^{-\frac{1}{2}+\kappa_W-q_W}d\tau] \\
&\quad -\left(q_W-2+3\overline{p}\right)\overline{p}^{q_W-1}\left(1-\overline{p}\right)^{-q_W} \\
&\quad \cdot[m_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{1}{2}+\kappa_W-q_W}\left(1-\tau\right)^{-\frac{3}{2}-\kappa_W+q_W}d\tau+n_W\int_{\widehat{p}}^{\overline{p}}\tau^{-\frac{1}{2}-\kappa_W-q_W}\left(1-\tau\right)^{-\frac{3}{2}+\kappa_W+q_W}d\tau]) \\
&\quad -\frac{\lambda}{\lambda+\gamma}\zeta_B^2\kappa_W\overline{p}^{-\frac{1}{2}-\kappa_W}\left(1-\overline{p}\right)^{-\frac{1}{2}+\kappa_W} \\
a_{55} &= -\frac{1}{2}\zeta_W^2\left(q_W-2+3\overline{p}\right)\overline{p}^{q_W-1}\left(1-\overline{p}\right)^{-q_W} \\
a_{56} &= -a_{55} \\
a_{57} &= \frac{1}{2}\zeta_W^2\left(-1-q_W+3\overline{p}\right)\overline{p}^{-q_W}\left(1-\overline{p}\right)^{q_W-1}
\end{aligned}
$$

$$
\begin{aligned}
a_{61} &= -\frac{1}{2}\zeta_B^2\left(-1-q_B+3\underline{p}\right)\underline{p}^{-q_B}\left(1-\underline{p}\right)^{q_B-1} \\
a_{62} &= \frac{1}{2}\zeta_B^2\left(q_B-2+3\underline{p}\right)\underline{p}^{q_B-1}\left(1-\underline{p}\right)^{-q_B} \\
a_{63} &= -a_{61} \\
a_{68} &= \frac{1}{2}\zeta_B^2\frac{1}{\sqrt{c_B\left(4+c_B\right)}}\cdot \\
&\quad (\left(-1-q_B+3\underline{p}\right)\underline{p}^{-q_B}\left(1-\underline{p}\right)^{q_B-1} \\
&\quad \cdot[m_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{3}{2}-\kappa_B+q_B}\left(1-\tau\right)^{-\frac{1}{2}+\kappa_B-q_B}d\tau+n_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{3}{2}+\kappa_B+q_B}\left(1-\tau\right)^{-\frac{1}{2}-\kappa_B-q_B}d\tau] \\
&\quad -\left(q_B-2+3\underline{p}\right)\underline{p}^{q_B-1}\left(1-\underline{p}\right)^{-q_B} \\
&\quad \cdot[m_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{1}{2}-\kappa_B-q_B}\left(1-\tau\right)^{-\frac{3}{2}+\kappa_B+q_B}d\tau+n_B\int_{\widehat{p}}^{\underline{p}}\tau^{-\frac{1}{2}+\kappa_B-q_B}\left(1-\tau\right)^{-\frac{3}{2}-\kappa_B+q_B}d\tau]) \\
&\quad +\frac{\lambda}{\lambda+\gamma}\zeta_W^2\kappa_B\underline{p}^{-\frac{1}{2}+\kappa_B}\left(1-\underline{p}\right)^{-\frac{1}{2}-\kappa_B}
\end{aligned}
$$

$$
\begin{aligned}
a_{76} &= \left(q_W-2+3\widehat{p}\right)\widehat{p}^{q_W-3}\left(1-\widehat{p}\right)^{-q_W-2} \\
a_{77} &= \left(-1-q_W+3\widehat{p}\right)\widehat{p}^{-2-q_W}\left(1-\widehat{p}\right)^{q_W-3} \\
a_{78} &= \left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B}\widehat{p}^{-\frac{5}{2}-\kappa_B}\left(1-\widehat{p}\right)^{-\frac{5}{2}+\kappa_B}\left(-\frac{3}{2}-\kappa_B+3\widehat{p}\right) \\
&\quad -\widehat{p}^{-\frac{5}{2}+\kappa_B}\left(1-\widehat{p}\right)^{-\frac{5}{2}-\kappa_B}\left(-\frac{3}{2}+\kappa_B+3\widehat{p}\right)
\end{aligned}
$$

$$
\begin{aligned}
a_{82} &= \left(q_B-2+3\widehat{p}\right)\widehat{p}^{q_B-3}\left(1-\widehat{p}\right)^{-q_B-2} \\
a_{83} &= \left(-1-q_B+3\widehat{p}\right)\widehat{p}^{-2-q_B}\left(1-\widehat{p}\right)^{q_B-3} \\
a_{84} &= \left(\frac{1-\overline{p}}{\overline{p}}\right)^{2\kappa_W}\widehat{p}^{-\frac{5}{2}+\kappa_W}\left(1-\widehat{p}\right)^{-\frac{5}{2}-\kappa_W}\left(-\frac{3}{2}+\kappa_W+3\widehat{p}\right) \\
&\quad -\widehat{p}^{-\frac{5}{2}-\kappa_W}\left(1-\widehat{p}\right)^{-\frac{5}{2}+\kappa_W}\left(-\frac{3}{2}-\kappa_W+3\widehat{p}\right)
\end{aligned}
$$

$$
\begin{aligned}
b_1 \;=\;& \frac{d_B}{\sqrt{c_B\left(4+c_B\right)}}\underline{p}^{q_B-2}\left(1-\underline{p}\right)^{-1-q_B}\int_0^{\underline{p}}\tau^{\psi_1-q_B}\left(1-\tau\right)^{q_B+\psi_2-1}d\tau \\
&-\frac{d_B}{\sqrt{c_B\left(4+c_B\right)}}\underline{p}^{-1-q_B}\left(1-\underline{p}\right)^{q_B-2}\int_0^{\underline{p}}\tau^{q_B+\psi_1-1}\left(1-\tau\right)^{\psi_2-q_B}d\tau \\
&+\frac{d_B}{\sqrt{c_B\left(4+c_B\right)}}[-\underline{p}^{q_B-2}\left(1-\underline{p}\right)^{-1-q_B}\int_{\widehat{p}}^{\underline{p}}\tau^{\psi_1-q_B}\left(1-\tau\right)^{\psi_2+q_B-1}d\tau \\
&+\underline{p}^{-1-q_B}\left(1-\underline{p}\right)^{q_B-2}\int_{\widehat{p}}^{\underline{p}}\tau^{\psi_1+q_B-1}\left(1-\tau\right)^{\psi_2-q_B}d\tau]
\end{aligned}
$$

$$
\begin{aligned}
b_2 \;=\;& \frac{d_W}{\sqrt{c_W\left(4+c_W\right)}}\overline{p}^{q_W-2}\left(1-\overline{p}\right)^{-1-q_W}\int_1^{\overline{p}}\tau^{\psi_1-q_W}\left(1-\tau\right)^{q_W+\psi_2-1}d\tau \\
&-\frac{d_W}{\sqrt{c_W\left(4+c_W\right)}}\overline{p}^{-1-q_W}\left(1-\overline{p}\right)^{q_W-2}\int_1^{\overline{p}}\tau^{q_W+\psi_1-1}\left(1-\tau\right)^{\psi_2-q_W}d\tau \\
&+\frac{d_W}{\sqrt{c_W\left(4+c_W\right)}}[-\overline{p}^{q_W-2}\left(1-\overline{p}\right)^{-1-q_W}\int_{\widehat{p}}^{\overline{p}}\tau^{\psi_1-q_W}\left(1-\tau\right)^{\psi_2+q_W-1}d\tau \\
&+\overline{p}^{-1-q_W}\left(1-\overline{p}\right)^{q_W-2}\int_{\widehat{p}}^{\overline{p}}\tau^{\psi_1+q_W-1}\left(1-\tau\right)^{\psi_2-q_W}d\tau]
\end{aligned}
$$

$$
\begin{aligned}
b_5 \;=\;& -\frac{1}{2}\zeta_W^2\frac{d_W}{\sqrt{c_W\left(4+c_W\right)}}\left(-1-q_W+3\overline{p}\right)\overline{p}^{-q_W}\left(1-\overline{p}\right)^{q_W-1}\int_{\widehat{p}}^{1}\tau^{\psi_1+q_W-1}\left(1-\tau\right)^{\psi_2-q_W}d\tau \\
&+\frac{1}{2}\zeta_W^2\frac{d_W}{\sqrt{c_W\left(4+c_W\right)}}\left(q_W-2+3\overline{p}\right)\overline{p}^{q_W-1}\left(1-\overline{p}\right)^{-q_W}\int_{\widehat{p}}^{1}\tau^{\psi_1-q_W}\left(1-\tau\right)^{\psi_2+q_W-1}d\tau
\end{aligned}
$$

$$
\begin{aligned}
b_6 \;=\;& \frac{1}{2}\zeta_B^2\frac{d_B}{\sqrt{c_B\left(4+c_B\right)}}\left(-1-q_B+3\underline{p}\right)\underline{p}^{-q_B}\left(1-\underline{p}\right)^{q_B-1}\int_0^{\widehat{p}}\tau^{\psi_1+q_B-1}\left(1-\tau\right)^{\psi_2-q_B}d\tau \\
&-\frac{1}{2}\zeta_B^2\frac{d_B}{\sqrt{c_B\left(4+c_B\right)}}\left(q_B-2+3\underline{p}\right)\underline{p}^{q_B-1}\left(1-\underline{p}\right)^{-q_B}\int_0^{\widehat{p}}\tau^{\psi_1-q_B}\left(1-\tau\right)^{q_B+\psi_2-1}d\tau.
\end{aligned}
$$

We also need to show that the determinant of matrix A is different from zero, in order to guarantee that a solution exists and is unique. Using Gaussian elimination one obtains

$$\det A = \det A^{*},$$

where

$$A^* =$$

$$\begin{bmatrix} a_{11} & a_{12} & -a_{11} & 0 & 0 & 0 & 0 & a_{18} \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{a^*_{62}a_{33}}{a_{32}} & -\frac{a^*_{62}a_{34}}{a_{32}} & 0 & 0 & 0 & \frac{a_{68}a_{11}-a_{61}a_{18}}{a_{11}} \\ 0 & 0 & 0 & a_{24} & a_{25} & -a_{25} & a_{27} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{46} & a_{47} & a_{48} \\ 0 & 0 & 0 & 0 & \frac{a_{55}a_{24}-a_{54}a_{25}}{a_{24}} & \frac{a_{54}a_{25}-a_{24}a_{55}}{a_{24}} & \frac{a_{57}a_{24}-a_{54}a_{27}}{a_{24}} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} & a_{78} \\ 0 & 0 & 0 & 0 & -\frac{a^*_{84}a_{25}}{a_{24}} & \frac{a^*_{84}a_{25}}{a_{24}} & -\frac{a^*_{84}a_{27}}{a_{24}} & \frac{a^*_{83}a_{32}}{a^*_{62}a_{33}}\left(a_{68} - \frac{a_{61}a_{18}}{a_{11}}\right) \end{bmatrix},$$

and

$$\begin{aligned} a^*_{62} &= -\frac{a_{61}a_{12}}{a_{11}} + a_{62} \\ a^*_{83} &= -\frac{a_{82}}{a_{32}}a_{33} + a_{83} \\ a^*_{84} &= -\frac{a_{34}}{a_{33}}a_{83} + a_{84}. \end{aligned}$$

Moreover let

$$K = \begin{bmatrix} a_{11} & a_{12} & -a_{11} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & -\frac{a^*_{62}}{a_{32}}a_{33} & -\frac{a^*_{62}}{a_{32}}a_{34} \\ 0 & 0 & 0 & a_{24} \end{bmatrix},$$

and

$$M = \begin{bmatrix} 0 & a_{46} & a_{47} & a_{48} \\ \frac{a_{55}a_{24}-a_{54}a_{25}}{a_{24}} & \frac{a_{54}a_{25}-a_{24}a_{55}}{a_{24}} & \frac{a_{57}a_{24}-a_{54}a_{27}}{a_{24}} & 0 \\ 0 & a_{76} & a_{77} & a_{78} \\ -\frac{a^*_{84}a_{25}}{a_{24}} & \frac{a^*_{84}a_{25}}{a_{24}} & -\frac{a^*_{84}a_{27}}{a_{24}} & \frac{a^*_{83}a_{32}}{a^*_{62}a_{33}}\left(a_{68} - \frac{a_{61}a_{18}}{a_{11}}\right) \end{bmatrix}.$$

Since A^* is a block matrix we can write

$$\det A = \det A^* = \det K \times \det M.$$

Since K is an (upper) triangular matrix, we have

$$\begin{aligned} \det K &= a_{11}a_{32}\left(-\frac{a^*_{62}}{a_{32}}a_{33}\right)a_{24} \\ &= \left(a_{61}a_{12} - a_{11}a_{62}\right)a_{33}a_{24}. \end{aligned}$$

Moreover using Gaussian elimination once again

$$\det M =$$

$$= \det \begin{bmatrix} -\frac{a_{84}^* a_{25}}{a_{24}} & \frac{a_{84}^* a_{25}}{a_{24}} & -\frac{a_{84}^* a_{27}}{a_{24}} & -\frac{a_{61}a_{18}}{a_{11}} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} + a_{68} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} \\ 0 & a_{46} & a_{47} & a_{48} \\ 0 & 0 & \frac{a_{57}a_{25} - a_{27}a_{55}}{a_{25}} & -\frac{a_{61}a_{18}}{a_{11}} \frac{a_{21}^* a_{24}}{a_{84}^* a_{25}} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} + a_{68} \frac{a_{21}^* a_{24}}{a_{84}^* a_{25}} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} \\ 0 & 0 & 0 & \frac{a_{25}\alpha_{43}^*}{a_{57}a_{25} - a_{27}a_{55}} \frac{a_{21}^* a_{24}}{a_{84}^* a_{25}} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} \left(-\frac{a_{61}a_{18}}{a_{11}} + a_{68} \right) - \frac{a_{76}}{a_{46}} a_{48} + a_{78} \end{bmatrix},$$

where

$$a_{21}^* = -\frac{a_{54}}{a_{24}} a_{25} + a_{55}$$

$$\alpha_{43}^* = -\frac{a_{76}}{a_{46}} a_{47} + a_{77},$$

which is also an (upper) triangular matrix, so

$$\det M =$$

$$\begin{aligned} = \quad & -\frac{a_{84}^*}{a_{24}} a_{25} a_{46} \left(-\frac{a_{27}a_{55}}{a_{25}} + a_{57} \right) \\ & \left(\frac{a_{25}\alpha_{43}^*}{a_{57}a_{25} - a_{27}a_{55}} \frac{a_{21}^* a_{24}}{a_{84}^* a_{25}} \frac{a_{83}^* a_{32}}{a_{62}^* a_{33}} \left(-\frac{a_{61}a_{18}}{a_{11}} + a_{68} \right) - \frac{a_{76}}{a_{46}} a_{48} + a_{78} \right). \end{aligned}$$

Some algebra leads to

$$\det A = \det K \times \det M =$$

$$\begin{aligned} = \quad & (a_{76}a_{47} - a_{77}a_{46})(a_{55}a_{24} - a_{25}a_{54})(a_{83}a_{32} - a_{82}a_{33})(a_{68}a_{11} - a_{61}a_{18}) \\ & + (a_{61}a_{12} - a_{11}a_{62})(a_{84}a_{33} - a_{83}a_{34})(a_{27}a_{55} - a_{25}a_{57})(a_{76}a_{48} - a_{78}a_{46}). \end{aligned}$$

After substituting in for a_{ij}, and a substantial amount of algebra, one obtains that the necessary and sufficient condition for the existence and uniqueness of our solution is that the following expression is different from zero

$$
\begin{aligned}
&\widehat{p}^{-8}\left(1-\widehat{p}\right)^{-8}\\
&[\frac{1}{2}\zeta_B^2\frac{1-2q_B}{\sqrt{c_B\left(4+c_B\right)}}\underline{p}^{-2}\left(1-\underline{p}\right)^{-2}\\
&[m_B\frac{1}{-\frac{1}{2}+\kappa_B+q_B}\left[\left(\frac{1-\underline{p}}{\underline{p}}\right)^{-\frac{1}{2}+\kappa_B+q_B}-\left(\frac{1-\widehat{p}}{\widehat{p}}\right)^{-\frac{1}{2}+\kappa_B+q_B}\right]\\
&+n_B\frac{1}{-\frac{1}{2}-\kappa_B+q_B}\left[\left(\frac{1-\underline{p}}{\underline{p}}\right)^{-\frac{1}{2}-\kappa_B+q_B}-\left(\frac{1-\widehat{p}}{\widehat{p}}\right)^{-\frac{1}{2}-\kappa_B+q_B}\right]]\\
&+\frac{\lambda}{\lambda+\gamma}\zeta_W^2\kappa_B\underline{p}^{-\frac{3}{2}+\kappa_B-q_B}\left(1-\underline{p}\right)^{-\frac{5}{2}-\kappa_B+q_B}]\\
&[-\frac{1}{2}\zeta_W^2\frac{1-2q_W}{\sqrt{c_W\left(4+c_W\right)}}\overline{p}^{-2}\left(1-\overline{p}\right)^{-2}\\
&[m_W\frac{1}{\frac{1}{2}-\kappa_W-q_W}\left[\left(\frac{1-\overline{p}}{\overline{p}}\right)^{\frac{1}{2}-\kappa_W-q_W}-\left(\frac{1-\widehat{p}}{\widehat{p}}\right)^{\frac{1}{2}-\kappa_W-q_W}\right]\\
&+n_W\frac{1}{\frac{1}{2}+\kappa_W-q_W}\left[\left(\frac{1-\overline{p}}{\overline{p}}\right)^{\frac{1}{2}+\kappa_W-q_W}-\left(\frac{1-\widehat{p}}{\widehat{p}}\right)^{\frac{1}{2}+\kappa_W-q_W}\right]]\\
&-\frac{\lambda}{\lambda+\gamma}\zeta_B^2\kappa_W\overline{p}^{-\frac{5}{2}-\kappa_W+q_W}\left(1-\overline{p}\right)^{-\frac{3}{2}+\kappa_W-q_W}]\\
&-\frac{1}{4}\zeta_W^2\zeta_B^2\underline{p}^{-2}\left(1-\underline{p}\right)^{-2}\overline{p}^{-2}\left(1-\overline{p}\right)^{-2}\\
&\widehat{p}^{-8-q_B+q_W}\left(1-\widehat{p}\right)^{-8+q_B-q_W}[\left(\frac{1-\overline{p}}{\overline{p}}\right)^{2\kappa_W}\left(\frac{\widehat{p}}{1-\widehat{p}}\right)^{\kappa_W}\left(-\frac{1}{2}+\kappa_W+q_B\right)\\
&+\left(\frac{\widehat{p}}{1-\widehat{p}}\right)^{-\kappa_W}\left(-q_B+\frac{1}{2}+\kappa_W\right)]\\
&[\left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\kappa_B}\left(\frac{\widehat{p}}{1-\widehat{p}}\right)^{-\kappa_B}\left(\frac{1}{2}-\kappa_B-q_W\right)+\left(\frac{\widehat{p}}{1-\widehat{p}}\right)^{\kappa_B}\left(q_W-\frac{1}{2}-\kappa_B\right)].
\end{aligned}
$$

For reasonable parameter values, the above expression is different than zero.

3.2 Fat Right Tails

As explained in the derivation of the steady-state distribution above, a necessary condition for the mass of workers not to approach minus infinity in the $[0,\underline{p})$ interval is that $q_B+\psi_1>0$. Similarly we also require that $q_W+\psi_2>0$.

We are interested in the behavior of $f_B(p)$ as p approaches 0.

We transform the integrals in equation (C.20) of the paper. Let $\tau=ps$. Then we can rewrite the first integral in the equation (C.20) as

$$\begin{aligned}\int_0^p \tau^{\psi_1 - q_B} (1-\tau)^{q_B+\psi_2-1} d\tau &= \int_0^1 (ps)^{\psi_1-q_B} (1-ps)^{q_B+\psi_2-1} pds \\ &= p^{1+\psi_1-q_B} \int_0^1 s^{\psi_1-q_B} (1-ps)^{q_B+\psi_2-1} ds.\end{aligned}$$

Similarly the second integral can be rewritten as

$$\int_0^p \tau^{q_B+\psi_1-1} (1-\tau)^{\psi_2-q_B} d\tau = p^{q_B+\psi_1} \int_0^1 s^{q_B+\psi_1-1} (1-ps)^{\psi_2-q_B} ds.$$

Thus we can now rewrite equation (C.20) of the paper as

$$\begin{aligned}f_B(p) &= C_2^B p^{-1-q_B} (1-p)^{q_B-2} \\ &\quad - \frac{d_B}{\sqrt{c_B(4+c_B)}} p^{\psi_1-1} (1-p)^{-1-q_B} \int_0^1 s^{\psi_1-q_B} (1-ps)^{q_B+\psi_2-1} ds \\ &\quad + \frac{d_B}{\sqrt{c_B(4+c_B)}} p^{\psi_1-1} (1-p)^{q_B-2} \int_0^1 s^{q_B+\psi_1-1} (1-ps)^{\psi_2-q_B} ds.\end{aligned}$$

Note that given the restriction $q_B + \psi_1 > 0$, it is true that $-1 - q_B < \psi_1 - 1$, so only the first term on the right hand side is quantitatively important as p approaches zero. Therefore a necessary and sufficient condition for the density of employed workers in occupation B to approach to zero as $p \to 0$ is that $-1 - q_B > 0$ which implies $\gamma > \zeta_B^2 \frac{\lambda+\gamma}{\delta_B+\lambda+\gamma}$. Put differently, the distribution of posterior beliefs features a fat Pareto-type tail if and only if $\gamma > \zeta_B^2 \frac{\lambda+\gamma}{\delta_B+\lambda+\gamma}$.

A similar derivation shows that f_W features a fat right tail, if and only if $\gamma > \zeta_W^2 \frac{\lambda+\gamma}{\delta_W+\lambda+\gamma}$.[4]

In Appendix C of the paper we derive conditions under which the wage distribution also features a fat right tail.

4 General Model Derivations

4.1 Belief Updating

An employed worker's belief follows a trivariate normal distribution with mean

[4]Note that these two conditions, along with the restrictions that $\psi_1 > -q_B$ and $\psi_2 > -q_W$ imply that $\psi_1, \psi_2 > 1$ (the birth distribution is unimodal).

$$\mu = \begin{bmatrix} \mu_1^k \\ \mu_2^k \\ \mu_3^k \end{bmatrix},$$

and variance-covariance matrix

$$\Sigma = \begin{bmatrix} \left(\tau_1^k\right)^2 & \rho_{12}^k \tau_1^k \tau_2^k & \rho_{13}^k \tau_1^k \tau_3^k \\ \rho_{12}^k \tau_1^k \tau_2^k & \left(\tau_2^k\right)^2 & \rho_{23}^k \tau_2^k \tau_3^k \\ \rho_{13}^k \tau_1^k \tau_3^k & \rho_{23}^k \tau_2^k \tau_3^k & \left(\tau_3^k\right)^2 \end{bmatrix}.$$

Consider a worker employed in occupation i. Then using Bayes' rule, his beliefs following a match output realization, $y_t^\kappa - g_{HC}(x)$, are updated as follows

$$\left(\mu_i^k\right)^* = \frac{\left(y_t^\kappa - g_{HC}(x)\right)\left(\tau_i^k\right)^2 + \sigma_i^2 \mu_i^k}{\left(\tau_i^k\right)^2 + \sigma_i^2}$$

$$\left(\mu_j^k\right)^* = \mu_j^k + \rho_{ji}^k \tau_j^k \tau_i^k n \frac{y_t^\kappa - g_{HC}(x) - \mu_i^k}{\left(\tau_i^k\right)^2 + \sigma_i^2}$$

$$\left(\tau_i^k\right)^* = \tau_i^k \sqrt{\frac{\sigma_i^2}{\left(\tau_i^k\right)^2 + \sigma_i^2}}$$

$$\left(\tau_j^k\right)^* = \tau_j^k \sqrt{\frac{\left(\tau_i^k\right)^2 \left(1 - \left(\rho_{ij}^k\right)^2\right) + \sigma_i^2}{\left(\tau_i^k\right)^2 + \sigma_i^2}}$$

$$\left(\rho_{ij}^k\right)^* = \rho_{ij}^k \frac{\sigma_i}{\sqrt{\left(\tau_i^k\right)^2 \left(1 - \left(\rho_{ij}^k\right)^2\right) + \sigma_i^2}}$$

$$\left(\rho_{jm}^k\right)^* = \frac{\left(\sigma_i^2 + \left(\tau_i^k\right)^2\right)\rho_{jm}^k - \left(\tau_i^k\right)^2 \rho_{ij}^k \rho_{im}^k}{\sqrt{\left(\left(\tau_i^k\right)^2 \left(1 - \left(\rho_{ij}^k\right)^2\right) + \sigma_i^2\right)\left(\left(\tau_i^k\right)^2 \left(1 - \left(\rho_{im}^k\right)^2\right) + \sigma_i^2\right)}},$$

where $i \neq j$ and $m \neq j$, while the $*$ superscript denotes updated values.

4.2 Wages

In this section we derive the wages for the general model introduced in Section 3.1 of the paper. The wage of employed workers is (re-)negotiated at the start of every period.

In the paper, we state the value functions for an employed and unemployed worker, equations (4) and (5).

The value of a firm is given by

$$\begin{aligned} J_i(\mu,\Sigma,x) &= \max\{I(\arg\max C(\mu,\Sigma,x)=i)[\mu_i+g_{HC}(x)-w_i^{NS}(\mu,\Sigma,x) \\ &+(1-\delta_i)\beta(1-\gamma)E_\mu J_i(\mu,\Sigma'(i),x')]+ \\ &(1-I(\arg\max C(\mu,\Sigma,x)=i))[(1-\eta_i\lambda)(\mu_i+g_{HC}(x) \\ &-w_i^{OTJS}(\mu,\Sigma,x)+(1-\delta_i)\beta(1-\gamma)E_\mu J_i(\mu,\Sigma'(i),x'))],0\}. \end{aligned}$$

It is easier to take cases in what follows, depending on whether is searching on-the-job or not.

We start with the case where he is looking for a job in occupation j, while employed in occupation i. We want to derive the wage the worker receives when the worker and firm decide to continue the match. Now the value functions become

$$\begin{aligned} V_i(\mu,\Sigma,x) &= \eta_i\lambda C(\mu,\Sigma,x)+ \\ &(1-\eta_i\lambda)(w_i^{OTJS}(\mu,\Sigma,x)+(1-\delta_i)\beta(1-\gamma)E_\mu V_i(\mu,\Sigma'(i),x') \\ &+\delta_i\beta(1-\gamma)E_\mu U(\mu,\Sigma'(i),x')) \end{aligned}$$

$$\begin{aligned} J_i(\mu,\Sigma,x) &= (1-\eta_i\lambda)(\mu_i+g_{HC}(x) \\ &-w_i^{OTJS}(\mu,\Sigma,x)+(1-\delta_i)\beta(1-\gamma)E_\mu J_i(\mu,\Sigma'(i),x')). \end{aligned}$$

Subtracting the worker's value of unemployment (equation (5) in the paper) from his value of being employed (equation (4) in the paper), and multiplying through by $1-q$ leads to

$$\begin{aligned} (1-q)(V_i(\mu,\Sigma,x)-U(\mu,\Sigma,x)) &= (1-q)\eta_i\lambda C(\mu,\Sigma,x) \\ &+(1-\eta_i\lambda)((1-q)w_i^{OTJS}(\mu,\Sigma,x) \\ &+(1-q)(1-\delta_i)\beta(1-\gamma)E_\mu V_i(\mu,\Sigma'(i),x') \\ &+(1-q)\delta_i\beta(1-\gamma)E_\mu U(\mu,\Sigma'(i),x'))-(1-q)U(\mu,\Sigma,x). \end{aligned}$$

Similarly multiplying the value of a filled job by q leads to

$$\begin{aligned} qJ_i(\mu,\Sigma,x) &= (1-\eta_i\lambda)(q\mu_i + qg_{HC}(x) \\ &\quad -qw_i^{OTJS}(\mu,\Sigma,x) + q(1-\delta_i)\beta(1-\gamma)E_\mu J_i(\mu,\Sigma'(i),x')). \end{aligned}$$

Subtracting the above two equations and using the surplus sharing condition (equation (6) in the paper), leads to

$$\begin{aligned} 0 &= (1-\eta_i\lambda)(q\mu_i + qg_{HC}(x) \\ &\quad -qw_i^{OTJS}(\mu,\Sigma,x) + q(1-\delta_i)\beta(1-\gamma)E_\mu J_i(\mu,\Sigma'(i),x')) \\ &\quad -(1-q)\eta_i\lambda C(\mu,\Sigma,x) - (1-\eta_i\lambda)((1-q)w_i^{OTJS}(\mu,\Sigma,x) \\ &\quad +(1-q)(1-\delta_i)\beta(1-\gamma)E_\mu V_i(\mu,\Sigma'(i),x') \\ &\quad +(1-q)\delta_i\beta(1-\gamma)E_\mu U(\mu,\Sigma'(i),x')) \\ &\quad +(1-q)U(\mu,\Sigma,x). \end{aligned}$$

From the surplus sharing condition, (equation (6) in the paper), we have

$$qE_\mu J_i(\mu,\Sigma'(i),x') - (1-q)E_\mu V_i(\mu,\Sigma'(i),x') = -(1-q)E_\mu U(\mu,\Sigma'(i),x'). \quad (13A)$$

So we can now rewrite the above as

$$\begin{aligned} (1-\eta_i\lambda)w_i^{OTJS}(\mu,\Sigma,x) &= (1-\eta_i\lambda)q(\mu_i + g_{HC}(x)) \\ &\quad +(1-\eta_i\lambda)(1-\delta_i)\beta(1-\gamma)(-(1-q)E_\mu U(\mu,\Sigma'(i),x')) \\ &\quad -(1-q)\eta_i\lambda C(\mu,\Sigma,x) \\ &\quad -(1-\eta_i\lambda)\delta_i\beta(1-\gamma)(1-q)E_\mu U(\mu,\Sigma'(i),x') \\ &\quad +(1-q)U(\mu,\Sigma,x) \end{aligned}$$

$$\begin{aligned} w_i^{OTJS}(\mu,\Sigma,x) &= q(\mu_i + g_{HC}(x)) + \frac{1-q}{1-\eta_i\lambda}U(\mu,\Sigma,x) \\ &\quad -(1-\delta_i)\beta(1-\gamma)(1-q)E_\mu U(\mu,\Sigma'(i),x') \\ &\quad -\delta_i\beta(1-\gamma)(1-q)E_\mu U(\mu,\Sigma'(i),x') \\ &\quad -\frac{\eta_i\lambda}{1-\eta_i\lambda}(1-q)C(\mu,\Sigma,x) \end{aligned}$$

$$
\begin{aligned}
w_i^{OTJS}\left(\mu,\Sigma,x\right) &= q(\mu_i+g_{HC}\left(x\right))+\frac{1-q}{1-\eta_i\lambda}U\left(\mu,\Sigma,x\right)\\
&-\beta\left(1-\gamma\right)\left(1-q\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right)\\
&-\frac{\eta_i\lambda}{1-\eta_i\lambda}\left(1-q\right)C\left(\mu,\Sigma,x\right).
\end{aligned}
$$

For the case of that the worker is not searching on-the-job, the value functions when the worker and firm choose not to dissolve the match are

$$
V_i\left(\mu,\Sigma,x\right)=w_i^{NS}\left(\mu,\Sigma,x\right)+\beta\left(1-\delta_i\right)\left(1-\gamma\right)E_\mu V_i\left(\mu,\Sigma'\left(i\right),x'\right)+\beta\delta_i\left(1-\gamma\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right)
$$

$$
J_i\left(\mu,\Sigma,x\right)=\mu_i+g_{HC}\left(x\right)-w_i^{NS}\left(\mu,\Sigma,x\right)+\left(1-\delta_i\right)\beta\left(1-\gamma\right)E_\mu J_i\left(\mu,\Sigma'\left(i\right),x'\right).
$$

As above, multiplying $V_i\left(\mu,\Sigma,x\right)$ and $U\left(\mu,\Sigma,x\right)$ by $1-q$ and multiplying $J_i\left(\mu,\Sigma,x\right)$ by q, subtracting and using the surplus sharing condition (equation (6) in the paper), leads to

$$
\begin{aligned}
0 &= q\left(\mu_i+g_{HC}\left(x\right)\right)-qw_i^{NS}\left(\mu,\Sigma,x\right)+q\left(1-\delta_i\right)\beta\left(1-\gamma\right)E_\mu J_i\left(\mu,\Sigma'\left(i\right),x'\right)\\
&-\left(1-q\right)w_i^{NS}\left(\mu,\Sigma,x\right)-\left(1-q\right)\beta\left(1-\delta_i\right)\left(1-\gamma\right)E_\mu V_i\left(\mu,\Sigma'\left(i\right),x'\right)\\
&-\left(1-q\right)\beta\delta_i\left(1-\gamma\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right)\\
&+\left(1-q\right)U\left(\mu,\Sigma,x\right).
\end{aligned}
$$

Using condition (13A) above, we obtain

$$
\begin{aligned}
w_i^{NS}\left(\mu,\Sigma,x\right) &= q\left(\mu_i+g_{HC}\left(x\right)\right)+\left(1-\delta_i\right)\beta\left(1-\gamma\right)\left(-\left(1-q\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right)\right)\\
&-\left(1-q\right)\beta\delta_i\left(1-\gamma\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right)\\
&+\left(1-q\right)U\left(\mu,\Sigma,x\right)
\end{aligned}
$$

$$
\begin{aligned}
w_i^{NS}\left(\mu,\Sigma,x\right) &= q\left(\mu_i+g_{HC}\left(x\right)\right)+\left(1-q\right)U\left(\mu,\Sigma,x\right)\\
&-\beta\left(1-\gamma\right)\left(1-q\right)E_\mu U\left(\mu,\Sigma'\left(i\right),x'\right).
\end{aligned}
$$

4.3 True Productivity Distribution

In this section we derive the distribution of the true productivity parameters.

We know that initial beliefs are determined as follows

$$v_1^k \sim N\left(\mu_1, \kappa_1^2\right)$$

$$v_2^k \sim N\left(\mu_2, \kappa_2^2\right)$$

$$v_3^k \sim N\left(\mu_3, \kappa_3^2\right),$$

and the true productivities are then drawn from

$$\mathbf{m}^k = \begin{bmatrix} m_1^k \\ m_2^k \\ m_3^k \end{bmatrix} \sim N\left(\begin{bmatrix} v_1^k \\ v_2^k \\ v_3^k \end{bmatrix}, \begin{bmatrix} \tau_1^2 & \rho_{12}\tau_1\tau_2 & \rho_{13}\tau_1\tau_3 \\ \rho_{12}\tau_1\tau_2 & \tau_2^2 & \rho_{23}\tau_2\tau_3 \\ \rho_{13}\tau_1\tau_3 & \rho_{23}\tau_2\tau_3 & \tau_3^2 \end{bmatrix}\right).$$

Let T be an upper-triangular matrix such that $T'T$ returns the above variance-covariance matrix. Moreover let Z be a 3x1 vector of i.i.d. standard normal random variables. Then we can write

$$\mathbf{m}^k = \mathbf{v}^k + T'Z,$$

where

$$\mathbf{v}^k = \begin{bmatrix} v_1^k \\ v_2^k \\ v_3^k \end{bmatrix}.$$

In our case let

$$T = \begin{bmatrix} h_1 & h_{12} & h_{13} \\ 0 & h_2 & h_{23} \\ 0 & 0 & h_3 \end{bmatrix},$$

and since

$$T'T = \begin{bmatrix} \tau_1^2 & \rho_{12}\tau_1\tau_2 & \rho_{13}\tau_1\tau_3 \\ \rho_{12}\tau_1\tau_2 & \tau_2^2 & \rho_{23}\tau_2\tau_3 \\ \rho_{13}\tau_1\tau_3 & \rho_{23}\tau_2\tau_3 & \tau_3^2 \end{bmatrix}$$

$$\begin{bmatrix} h_1^2 & h_1h_{12} & h_1h_{13} \\ h_1h_{12} & h_2^2 + h_{12}^2 & h_2h_{23} + h_{12}h_{13} \\ h_1h_{13} & h_2h_{23} + h_{12}h_{13} & h_3^2 + h_{13}^2 + h_{23}^2 \end{bmatrix} = \begin{bmatrix} \tau_1^2 & \rho_{12}\tau_1\tau_2 & \rho_{13}\tau_1\tau_3 \\ \rho_{12}\tau_1\tau_2 & \tau_2^2 & \rho_{23}\tau_2\tau_3 \\ \rho_{13}\tau_1\tau_3 & \rho_{23}\tau_2\tau_3 & \tau_3^2 \end{bmatrix},$$

which implies

$$h_1 = \tau_1$$

$$h_{12} = \rho_{12}\tau_2$$

$$h_{13} = \rho_{13}\tau_3$$

$$\begin{aligned} h_2^2 &= \tau_2^2 - h_{12}^2 \\ &= \tau_2^2 - \rho_{12}^2\tau_2^2 \end{aligned}$$

$$\begin{aligned} h_2 &= \sqrt{\tau_2^2 - \rho_{12}^2\tau_2^2} \\ &= \tau_2\sqrt{(1-\rho_{12}^2)} \end{aligned}$$

$$\begin{aligned} h_2 h_{23} + h_{12}h_{13} &= \rho_{23}\tau_2\tau_3 \Leftrightarrow \\ h_{23}\tau_2\sqrt{(1-\rho_{12}^2)} + \rho_{12}\tau_2\rho_{13}\tau_3 &= \rho_{23}\tau_2\tau_3 \Leftrightarrow \\ h_{23} &= \frac{\tau_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)}{\sqrt{(1-\rho_{12}^2)}} \end{aligned}$$

$$\begin{aligned} h_3^2 + h_{13}^2 + h_{23}^2 &= \tau_3^2 \Leftrightarrow \\ h_3^2 + \rho_{13}^2\tau_3^2 + \frac{\tau_3^2\left(\rho_{23}-\rho_{12}\rho_{13}\right)^2}{(1-\rho_{12}^2)} &= \tau_3^2 \Leftrightarrow \\ h_3 &= \tau_3\sqrt{1-\rho_{13}^2 - \frac{\left(\rho_{23}-\rho_{12}\rho_{13}\right)^2}{(1-\rho_{12}^2)}}. \end{aligned}$$

So we can write the trivariate distribution as

$$m_1^k = v_1^k + \tau_1 Z_1$$

$$m_2^k = v_2^k + \tau_2\left(\rho_{12}Z_1 + \left(1-\rho_{12}^2\right)^{1/2} Z_2\right)$$

$$m_3^k = v_3^k + \rho_{13}\tau_3 Z_1 + \frac{\tau_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)}{\sqrt{(1-\rho_{12}^2)}} Z_2 + \tau_3\sqrt{1-\rho_{13}^2 - \frac{\left(\rho_{23}-\rho_{12}\rho_{13}\right)^2}{(1-\rho_{12}^2)}} Z_3,$$

and substituting in for v_i^k leads to

$$m_1^k = \mu_1 + \kappa_1\Psi_1 + \tau_1 Z_1$$

$$m_2^k = \mu_2 + \kappa_2\Psi_2 + \tau_2\left(\rho_{12}Z_1 + \left(1-\rho_{12}^2\right)^{1/2} Z_2\right)$$

$$m_3^k = \mu_3 + \kappa_3\Psi_3 + \rho_{13}\tau_3 Z_1 + \frac{\tau_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)}{\sqrt{(1-\rho_{12}^2)}}Z_2 + \tau_3 Z_3\sqrt{1-\rho_{13}^2-\frac{\left(\rho_{23}-\rho_{12}\rho_{13}\right)^2}{(1-\rho_{12}^2)}}.$$

Given that Z_3 and Ψ_3 are i.i.d. standard normal we can write the above as

$$m_1^k = \mu_1 + \kappa_1\Psi_1 + \tau_1 Z_1$$

$$m_2^k = \mu_2 + \kappa_2\Psi_2 + \tau_2\rho_{12}Z_1 + \tau_2\left(1-\rho_{12}^2\right)^{1/2} Z_2$$

$$m_3^k = \mu_3 + \Psi_4\sqrt{\left(\kappa_3^2 + \tau_3^2\left(1-\rho_{13}^2-\frac{\left(\rho_{23}-\rho_{12}\rho_{13}\right)^2}{(1-\rho_{12}^2)}\right)\right)} + \rho_{13}\tau_3 Z_1 + \frac{\tau_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)}{\sqrt{(1-\rho_{12}^2)}}Z_2,$$

where Ψ_4 also follows a $N(0,1)$.

We rewrite the above system as

$$\begin{aligned} m_1^k &= \mu_1 + \lambda_1 Z_1 + \xi_1\Psi_1 \\ m_2^k &= \mu_2 + \lambda_2 Z_1 + \lambda_3 Z_2 + \xi_2\Psi_2 \\ m_3^k &= \mu_3 + \lambda_4 Z_1 + \lambda_5 Z_2 + \xi_3\Psi_4, \end{aligned}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \xi_1, \xi_2$ and ξ_3 are appropriately defined and $Z_1, Z_2, \Psi_1, \Psi_2, \Psi_4$ are i.i.d. standard normals. We next show that the above system can be written in the form of a trivariate normal distribution.

The moment generating function of $\left(m_1^k, m_2^k, m_3^k\right)$ is given by

$$M_{m_1^k,m_2^k,m_3^k}\left(t_1,t_2,t_3\right)$$

$$= E\left(e^{t_1 m_1^k} e^{t_2 m_2^k} e^{t_3 m_3^k}\right)$$

$$= E\left(e^{(\mu_1+\lambda_1 Z_1+\xi_1\Psi_1)t_1} e^{(\mu_2+\lambda_2 Z_1+\lambda_3 Z_2+\xi_2\Psi_2)t_2} e^{(\mu_3+\lambda_4 Z_1+\lambda_5 Z_2+\xi_3\Psi_4)t_3}\right)$$

$$= E\left(e^{(\mu_1+\xi_1\Psi_1)t_1}\right) E\left(e^{(\mu_2+\xi_2\Psi_2)t_2}\right) E\left(e^{(\mu_3+\xi_3\Psi_4)t_3}\right) E\left(e^{(\lambda_1 t_1+\lambda_2 t_2+\lambda_4 t_3)Z_1}\right) E\left(e^{(\lambda_3 t_2+\lambda_5 t_3)Z_2}\right)$$

$$\begin{aligned} = \quad & \exp[\mu_1 t_1 + \frac{\xi_1^2 t_1^2}{2} + \mu_2 t_2 + \frac{\xi_2^2 t_2^2}{2} + \mu_3 t_3 + \frac{\xi_3^2 t_3^2}{2} + \frac{\lambda_1^2 t_1^2}{2} + \frac{\lambda_2^2 t_2^2}{2} + \frac{\lambda_4^2 t_3^2}{2} \\ & + \frac{2\lambda_1\lambda_2 t_1 t_2}{2} + \frac{2\lambda_1\lambda_4 t_1 t_3}{2} + \frac{2\lambda_2\lambda_4 t_2 t_3}{2} + \frac{\lambda_3^2 t_2^2}{2} + \frac{\lambda_5^2 t_3^2}{2} + \frac{2\lambda_3\lambda_5 t_2 t_3}{2}] \end{aligned}$$

$$= \exp[\mu_1 t_1 + \mu_2 t_2 + \mu_3 t_3 + \frac{\left(\xi_1^2 + \lambda_1^2\right) t_1^2}{2} + \frac{\left(\xi_2^2 + \lambda_2^2 + \lambda_3^2\right) t_2^2}{2} + \frac{\left(\xi_3^2 + \lambda_4^2 + \lambda_5^2\right) t_3^2}{2}$$
$$+ \frac{2\lambda_1\lambda_2 t_1 t_2}{2} + \frac{2\lambda_1\lambda_4 t_1 t_3}{2} + \frac{2\left(\lambda_2\lambda_4 + \lambda_3\lambda_5\right) t_2 t_3}{2}].$$

This last expression however is the moment generating function of a trivariate normal distribution with mean

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix},$$

and variance-covariance matrix

$$\begin{bmatrix} \xi_1^2 + \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_4 \\ \lambda_1\lambda_2 & \xi_2^2 + \lambda_2^2 + \lambda_3^2 & \lambda_2\lambda_4 + \lambda_3\lambda_5 \\ \lambda_1\lambda_4 & \lambda_2\lambda_4 + \lambda_3\lambda_5 & \xi_3^2 + \lambda_4^2 + \lambda_5^2 \end{bmatrix}.$$

Consequently, since the moment generating function, if it exists, determines the distribution uniquely, we can substitute in for the ξ_i and λ_i using our model's notation, to find that the true productivities are drawn from

$$N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \begin{bmatrix} \kappa_1^2 + \tau_1^2 & \rho_{12}\tau_1\tau_2 & \rho_{13}\tau_1\tau_3 \\ \rho_{12}\tau_1\tau_2 & \kappa_2^2 + \tau_2^2 & \rho_{23}\tau_2\tau_3 \\ \rho_{13}\tau_1\tau_3 & \rho_{23}\tau_2\tau_3 & \kappa_3^2 + \tau_3^2 \end{bmatrix}\right).$$

4.4 Wages and Value Functions for Older Workers

In this section derive the wage and the value functions for workers with more than 20 years of labor market experience. These workers have no incentive to search on the job and in the exogenous displacement they return to occupation i.

We have

$$V_i(\mu, \Sigma, x) = w_i^{NS}(\mu, \Sigma, x) + \beta(1-\delta_i)(1-\gamma) E_\mu V_i(\mu, \Sigma'(i), x') + \beta\delta_i(1-\gamma) E_\mu U(\mu, \Sigma'(i), x')$$

$$U(\mu, \Sigma, x) = b_u + \beta(1-\gamma)\lambda V_i(\mu, \Sigma, x) + \beta(1-\gamma)(1-\lambda) U(\mu, \Sigma, x).$$

Moreover, since by assumption the worker has learned his type, there is no more belief updating and also the worker does not accumulate more human capital accumulation[5], so we can rewrite the value of an employed worker as

[5] Bagger et al. (2013) find that the accumulation of human capital slows down to a minimum after 20 years of labor market experience.

$$V_i(\mu,\Sigma,x) = w_i^{NS}(\mu,\Sigma,x) + \beta(1-\delta_i)(1-\gamma)V_i(\mu,\Sigma,x) + \beta\delta_i(1-\gamma)U(\mu,\Sigma,x).$$

Similarly the value of a firm employing a worker with more than 20 years of labor market experience is given by

$$J_i(\mu,\Sigma,x) = \mu_i + g_{HC}(x) - w_i^{NS}(\mu,\Sigma,x) + (1-\delta_i)\beta(1-\gamma)J_i(\mu,\Sigma,x).$$

As before multiplying $V_i(\mu,\Sigma,x)$ and $U(\mu,\Sigma,x)$ by $1-q$ and multiplying $J_i(\mu,\Sigma,x)$ by q, subtracting and using the surplus sharing condition (equation (6) in the paper), leads to

$$\begin{aligned} 0 \;=\; & q(\mu_i + g_{HC}(x)) - qw_i^{NS}(\mu,\Sigma,x) + q(1-\delta_i)\beta(1-\gamma)J_i(\mu,\Sigma,x) \\ & -(1-q)w_i^{NS}(\mu,\Sigma,x) - (1-q)\beta(1-\delta_i)(1-\gamma)V_i(\mu,\Sigma,x) - (1-q)\beta\delta_i(1-\gamma)U(\mu,\Sigma,x) \\ & +(1-q)U(\mu,\Sigma,x). \end{aligned}$$

Moreover, since from the surplus sharing condition we have

$$\begin{aligned} & q(1-\delta_i)\beta(1-\gamma)J_i(\mu,\Sigma,x) - (1-q)\beta(1-\delta_i)(1-\gamma)V_i(\mu,\Sigma,x) \\ = \; & -(1-q)\beta(1-\delta_i)(1-\gamma)U(\mu,\Sigma,x), \end{aligned}$$

and

$$\begin{aligned} w_i^{NS}(\mu,\Sigma,x) \;=\; & q(\mu_i + g_{HC}(x)) \\ & -(1-q)\beta\delta_i(1-\gamma)U(\mu,\Sigma,x) \\ & +(1-q)U(\mu,\Sigma,x) - (1-q)\beta(1-\delta_i)(1-\gamma)U(\mu,\Sigma,x) \end{aligned}$$

$$w_i^{NS}(\mu,\Sigma,x) = q(\mu_i + g_{HC}(x)) + (1-q)(1-\beta(1-\gamma))U(\mu,\Sigma,x). \tag{14A}$$

Moreover from the value of unemployment we have

$$U(\mu,\Sigma,x) = \frac{b_u}{1-\beta(1-\gamma)(1-\lambda)} + \frac{\beta(1-\gamma)\lambda}{1-\beta(1-\gamma)(1-\lambda)}V_i(\mu,\Sigma,x).$$

Using the derived wage equation we value of unemployment above, we can solve out for the value and wage of workers with more than 20 years of labor market experience.

In particular, substituting out for the wage leads to

$$V_i(\mu,\Sigma,x)=\frac{w_i^{NS}(\mu,\Sigma,x)}{1-\beta(1-\delta_i)(1-\gamma)}+\frac{\beta\delta_i(1-\gamma)}{1-\beta(1-\delta_i)(1-\gamma)}U(\mu,\Sigma,x)$$

$$V_i(\mu,\Sigma,x)=\frac{q(\mu_i+g_{HC}(x))}{1-\beta(1-\delta_i)(1-\gamma)}+\frac{\beta\delta_i(1-\gamma)+(1-q)(1-\beta(1-\gamma))}{1-\beta(1-\delta_i)(1-\gamma)}U(\mu,\Sigma,x).$$

Substituting out for the value of unemployment leads to

$$\begin{aligned}V_i(\mu,\Sigma,x) &= \frac{q(\mu_i+g_{HC}(x))}{1-\beta(1-\delta_i)(1-\gamma)}+\frac{\beta\delta_i(1-\gamma)+(1-q)(1-\beta(1-\gamma))}{1-\beta(1-\delta_i)(1-\gamma)}\frac{b_u}{1-\beta(1-\gamma)(1-\lambda)}\\ &\quad+\frac{\beta\delta_i(1-\gamma)+(1-q)(1-\beta(1-\gamma))}{1-\beta(1-\delta_i)(1-\gamma)}\frac{\beta(1-\gamma)\lambda}{1-\beta(1-\gamma)(1-\lambda)}V_i(\mu,\Sigma,x)\end{aligned}$$

$$\begin{aligned}V_i(\mu,\Sigma,x) &= \frac{q(\mu_i+g_{HC}(x))}{1-\beta(1-\delta_i)(1-\gamma)}k_i\\ &\quad+\frac{\beta\delta_i(1-\gamma)+(1-q)(1-\beta(1-\gamma))}{1-\beta(1-\delta_i)(1-\gamma)}\frac{b_u}{1-\beta(1-\gamma)(1-\lambda)}k_i,\end{aligned} \tag{15A}$$

where

$$k_i=\left(1-\frac{(1-q)(1-\beta(1-\gamma))+\beta\delta_i(1-\gamma)}{1-\beta(1-\delta_i)(1-\gamma)}\frac{\beta(1-\gamma)\lambda}{1-\beta(1-\gamma)(1-\lambda)}\right)^{-1}.$$

Going back to the wage equation (14A), and substituting in for $U(\mu,\Sigma,x)$, leads to

$$\begin{aligned}w_i^{NS}(\mu,\Sigma,x) &= q(\mu_i+g_{HC}(x))+(1-q)(1-\beta(1-\gamma))\frac{b_u}{1-\beta(1-\gamma)(1-\lambda)}\\ &\quad+\frac{\beta(1-\gamma)\lambda(1-q)(1-\beta(1-\gamma))}{1-\beta(1-\gamma)(1-\lambda)}V_i(\mu,\Sigma,x).\end{aligned}$$

Substituting in for the $V_i(\mu,\Sigma,x)$ using the expression that we found above gives us the wage function for workers with more than 20 years of labor market experience. as a function of parameters

$$
\begin{aligned}
w_i^{NS}(\mu,\Sigma,x) &= q\left(\mu_i+g_{HC}(x)\right)+(1-q)\left(1-\beta(1-\gamma)\right)\frac{b_u}{1-\beta(1-\gamma)(1-\lambda)} \qquad (16A)\\
&+\frac{\beta(1-\gamma)\lambda(1-q)\left(1-\beta(1-\gamma)\right)}{1-\beta(1-\gamma)(1-\lambda)}\frac{q\left(\mu_i+g_{HC}(x)\right)}{1-\beta(1-\delta_i)(1-\gamma)}k_i\\
&+\frac{\beta(1-\gamma)\lambda(1-q)\left(1-\beta(1-\gamma)\right)}{\left(1-\beta(1-\gamma)(1-\lambda)\right)^2}\frac{\beta\delta_i(1-\gamma)+(1-q)\left(1-\beta(1-\gamma)\right)}{1-\beta(1-\delta_i)(1-\gamma)}b_u k_i.
\end{aligned}
$$

For a given set of parameters values one can now back out the implied occupational choices (using the value function equation (15A)) and the associated wages (using equation (16A) above).

5 Occupation Partition

The 1996 panel uses the 1990 occupation classification system, which is based on the 1980 SOC manual.

For high school graduates, we partition the three occupations as follows:

White-collar jobs (occupation 1) include those under the headers:

Executive, Administrative and Managerial Occupations (003-022), Management Related Occupations(023-037), Engineers, Architects, and Surveyors (043-199), Technicians and Related Support Occupations (203-235), Sales Occupations (243-285), Administrative Support Occupations Including Clerical (303-389), Private Household Occupations (403-407), Protective Service Occupations (413-427), Service Occupations, Except Protective and Household (433-469).

Blue-collar jobs that involve precision production and repairs (occupation 2) include those under the headers:

Farm operators and Managers (473-476), Other Agricultural and Related Occupations (477-489), Forestry and Logging Occupations (494-496), Fisher, Hunters and Trappers (497-499), Mechanics and Repairers (503-549), Construction Trades (553-599), Extractive Occupations (613-617), Precision Production Occupations (628-694), Plant and System Operators (694-699)

Blue-collar jobs that involve operators and laborers (occupation 3) include those under the headers:

Machine Operators, Assemblers and Inspectors (703-799), Transportation and Material Moving Occupations (803-859), Handlers, Equipment Cleaners, Helpers and Laborers (864-889).

We also run the calibration for high school graduates, by grouping occupations by first index of the occupational code (first three, middle three, last three). In that case we partition the three occupations as follows:

Occupation 1 includes those under the headers:

Executive, Administrative and Managerial Occupations (003-022), Management Related Occupations(023-037), Engineers, Architects, and Surveyors (043-199), Technicians and Related Support Occupations (203-235), Sales Occupations (243-285).

Occupation 2 includes those under the headers:

Administrative Support Occupations Including Clerical (303-389), Private Household Occupations (403-407), Protective Service Occupations (413-427), Service Occupations, Except Protective and Household (433-469), Farm operators and Managers (473-476), Other Agricultural and Related Occupations (477-489), Forestry and Logging Occupations (494-496), Fisher, Hunters and Trappers (497-499), Mechanics and Repairers (503-549), Construction Trades (553-599).

Occupation 3 includes those under the headers:

Extractive Occupations (613-617), Precision Production Occupations (628-694), Plant and System Operators (694-699), Machine Operators, Assemblers and Inspectors (703-799), Transportation and Material Moving Occupations (803-859), Handlers, Equipment Cleaners, Helpers and Laborers (864-889).

For workers with some college and college education, we partition the three occupations as follows:

White-collar jobs (occupation 1) include those under the headers:

Executive, Administrative and Managerial Occupations (003-022), Management Related Occupations(023-037), Engineers, Architects, and Surveyors (043-199), Technicians and Related Support Occupations (203-235).

Blue-collar jobs (occupation 2) include those under the headers:

Farm operators and Managers (473-476), Other Agricultural and Related Occupations (477-489), Forestry and Logging Occupations (494-496), Fisher, Hunters and Trappers (497-499), Mechanics and Repairers (503-549), Construction Trades (553-599), Extractive Occupations (613-617), Precision Production Occupations (628-694), Plant and System Operators (694-699), Machine Operators, Assemblers and Inspectors (703-799), Transportation and Material Moving Occupations (803-859), Handlers, Equipment Cleaners, Helpers and Laborers (864-889).

Pink collar jobs (occupation 3) include those under the headers:

Sales Occupations (243-285), Administrative Support Occupations Including Clerical (303-389), Private Household Occupations (403-407), Protective Service Occupations (413-

427), Service Occupations, Except Protective and Household (433-469).

6 Job-Finding Rate Estimation

To estimate the job-finding rate, we follow Shimer (2012). In particular, in the model we assume that job offers arrive at a Poisson rate λ. The probability that a worker has not received a job offer by time t (conditional on being unemployed at $t = 0$) is given by $\Pr(\text{no offers by } t) \equiv e^{-\lambda t}$. If we set $t = 1$ to denote one month and let Λ denote the probability that the worker received a job offer within the month, then $\lambda = -\ln(1 - \Lambda)$. Therefore we need to back out the Λ from the data.

Let u_t denote the number of unemployed workers at the end of month t and u_t^s the number of workers who at time t have been unemployed for less than a month; then as in Shimer (2012) we have

$$u_{t+1} = (1 - \Lambda_t)\, u_t + u_{t+1}^s,$$

which implies

$$\Lambda_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}.$$

Since the SIPP provides weekly reports of the labor market status of each worker, it is straightforward to calculate both u_t and u_t^s for all t. We calculate the job-finding rates by dropping all observations that do not have a complete employment history. Furthermore we use individuals who report searching actively for employment while unemployed.

Now we can compute the 47 monthly job-finding probabilities.

For workers with a college degree and some college, the estimated job-finding rate implies an average unemployment duration of less than 4 months (2.96 and 2.92 months respectively), which is the length of a period in our setup. Given that a worker who becomes unemployed spends at least one period in that state, we set λ equal to 1 in both cases.

7 Calibration Details

The procedure seeks to minimize the distance between simulated and empirical moments, first for older workers and then for younger workers. The weighting matrix in each case is given by the inverse of the variance-covariance matrix of the empirical moments. As these steps are performed iteratively, errors in the calculation of parameters ϕ_1 are inevitably passed to the second step. Simulation of the model in the second step involves solving for optimal worker behavior (which occupation to search for when unemployed, whether to engage in on-the-job search and whether to quit his current occupation). The relevant state space, which includes

125 combination of beliefs and 13 experience levels for each of the three occupations, contains 274,625 elements; this implies that each of the three transition matrices contains 34,328,125 elements. For the actual simulation, there are no restrictions on worker beliefs (i.e. they do not lie on a grid), nor on the experience levels, so the discretization does not affect the calculation of accumulated worker experience, nor wages, only worker decisions. Moreover, the restriction on the experience levels concerns the second step where we are interested in the moments of the wage distributions of younger workers.

References

[1] Bagger, J., F. Fontaine, F. Postel-Vinay and J.-M. Robin (2013): "Tenure, Experience, Human Capital and Wages: A Tractable Equilibrium Search Model of Wage Dynamics," *American Economic Review*, forthcoming.

[2] Shimer, R. (2012): "Reassessing the Ins and Outs of Unemployment," *Review of Economic Dynamics*, 15(2), 127-148.